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Marshall, Consumer Surplus, and the Marginal Utility of Money

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The major theoretic objections to Marshall's consumer surplus measure are those which follow from the unreasonable assumption and implications of constant marginal utility of money which is required for the measure to be meaningful. Samuelson showed in his *Foundations* that constancy must refer to something like real income rather than nominal income and then demonstrated that it would also imply unitary price elasticity of demand for any commodity, a relationship that simply does not hold in real markets.[1] But although we recognize that this violation of reality compromises the constancy assumption, it is often argued that marginal utility is approximately constant for small changes in income, and that the zero income effect which follows from a price fall and logically requires that assumption is argued to be approximately correct much of the time; thus the implied constancy can be accepted after all and, with it, the consumer surplus measure of welfare gain.

In this note we will argue, to the contrary, that Marshall did not hold, and did not need, the assumption of constant marginal utility of money and that consumer surplus is not a good measure of welfare gain. Moreover, I suspect that Marshall would agree with this criticism on empirical grounds.

In order that the "Marshallian consumer surplus," the area under the market demand curve between prices p_1 and p_2, be an acceptable measure of the change in social welfare accompanying the price change it is necessary that the marginal utility of money (income) be constant. This is the case for two reasons:

(i) To compare his own welfare before and after changes in price and consumption each individual consumer must have an unvarying measure denominated in a fixed unit, the utility of one extra dollar; and

(ii) The market demand curve sums individual demand curves horizontally. Thus, if the area under the curve is to be a meaningful measure of welfare change, the utility of each dollar of difference between the demand price for any infra-marginal unit of consumption and the equilibrium market price must be the same for any two individuals. Since their incomes may be very different, this can be the case only if the utility of each extra dollar is constant for all levels of income, i.e., if the marginal utility of money is constant.

The notion of constant marginal utility of money conflicts with our basic postulate that the utility of any commodity increases at a decreasing rate as the volume of that commodity increases, i.e., that the marginal utility is decreasing. Accordingly, Marshall is faulted for holding to this assumption; indeed, many project evaluation manuals and studies fault Marshall for holding this unreasonable assumption (and its implied zero income effect on consumption of a good following a price change) to argue that, in the example at hand, marginal utility does not decrease very much and the income effect is small so that the measuring rod is virtually constant and

[1]Samuelson, of course, objected also on the grounds that it was an attempt to cardinalize what were essentially ordinal indifference relationships. However, the development of the indirect utility function overcomes this objection.

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there is no possibility of double counting. Frequently there is an appeal to a well-known passage from Hicks.[2]

But the truth is that Marshall did not accept the constancy hypothesis, either for an individual or for consumers at different income levels. Since this conflicts with the usual view of things I would like to offer the following citations testifying to this view. But first some differences in terminology which have drawn too little attention should be mentioned.

Marshall employed three terms to mean two different things: on the one hand he uses "marginal value of money" (1926 p. 109) and "marginal degree of utility of money" (p. 691) to mean what we call "marginal utility of money." ("If m is the amount of money or general purchasing power at a person's disposal at any time, and μ represents its total utility to him, then $d\mu/dm$ represents the marginal degree of utility to him." (p. 690). On the other hand, "If u be the total utility of an amount x of a commodity to a given person at a given time, then marginal utility is measured by $du/dx\ \delta x$." (p. 690). Thus, Marshall's marginal utility of money is the marginal degree of utility of money multiplied by a differential.

Perhaps the clearest statement of behaviour of margina degree of utility is on p. 690:

> "Every increase in his means diminishes the marginal degree of utility of money to him [i.e. our marginal utility]; that is, $d^2\mu/dm^2$ is always negative."

Further, discussing the use of the consumer surplus triangle, he argues (p. 109):

> The substance of our argument would not be affected if we took account of the fact that, the more a person spends on anything the less power he retains of purchasing more of it or of other things, and the greater is the value of money to him (in technical language every fresh expenditure increases the marginal value of money to him). But though its substance would not be altered, its form would be made more intricate without any corresponding gain; for there are very few practical problems, in which the correction to be made under this head would be of any importance[2] (p. 109).

The last paragraph in the note cited reads (p. 109):

> As is indicated in Note VI in the Mathematical Appendix, formal account could be taken of changes in the marginal utility of money, if it were desired to do so. If we attempted to add together the total utilities of all commodities, we should be bound to do so: that task is however impracticable.

And the citation referred to in the Mathematical Appendix tells us (p. 693):

> It should be noted that, in the discussion of consumers' surplus, we assume that the marginal utility of money to the individual purchaser is the same throughout. Strictly speaking we ought to take account of the fact that if he spent less on tea, marginal utility of money to him would be less than it is, and he would get an element of consumers' surplus from buying other things at prices which now yield him no such rent. But these changes of consumers' rent (being of the second order of smallness) may be neglected, on the assumption, which underlies our whole reasoning, that his expenditure on any one thing, as, for instance, tea, is only a small part of his whole expenditure.[3]

[2]"What in the light of this approach, we have been trying to do is to establish more precisely than Marshall thought necessary, the conditions needed for the Marshall measure to be a good measure. And, so considered, the result of our inquiry is very simple. In order that the Marshall measure of consumer's surplus should be a good measure, one thing alone is needful—that the income effect should be small." We note that when objecting to the "Marshallian measure" and offering his other consumer supluses, Hicks stressed that each answered a different question. In general, the compensating variation and the compensated demand curve have come to be accepted as the main alternatives to the "Marshallian approach." Interestingly, concerning the adequacy of the compensating and equivalent variation approach as general welfare measures, in the sense of permitting comparisons between any two points on the budget space, Chipman and Moore (1980) have recently demonstrated that equivalent variation is an acceptable measure in a greater variety of circumstances than compensating variation.

[3]It is also interesting to note here that unitary elasticity of demand follows much more straightforwardly from Marshall than is sometimes thought. Thus, since marginal degree of utility (our marginal utility) *is* assumed to be decreasing, the only way a situation can eventuate in which money is an unchanged measuring rod is if the amount of money at the consumers disposal does not change. That is, the differential in income resulting from the price change must be zero, implying that the amount spent on the commodity following the price change must be unchanged, i.e., its price elasticity must be unitary. This last result—that the differential be zero—also implies that what Marshall called the "marginal utility of money" be zero even though the marginal degree" (our marginal utility, $d\mu/dm$) be positive.

Thus, Marshall certainly acknowledged that the marginal utility of income (today's terminology) falls if the price of a commodity with price-inelastic demand declines; in fact it was a basic tenet! He believed that, for the most part, there was little to be gained from introducing such refinements as would be required to allow for inconstant marginal utility for a broad range of commodities, a position not very different from Hicks's. Both accept the "Marshallian triangle" as a measure of welfare change for an individual consumer, but as is clear from the last quotation, only so long as near constancy is reasonable.

We have so far focused on the implications of constancy and non-constancy for the individual consumer and the use of the Marshallian triangle to infer changes in his welfare (even though Marshall sometimes placed the apostrophe after the "s" as in the last citation). What of constancy and the community?

Part of the prolixity which readers of Marshall have sometimes found vexatious is caused by his numerous qualifications concerning behaviour of consumers—that the middle-income or rich individual may have different tastes etc.

For example, the marginal utility of twopence is recognized to be different for a rich man and a poor man (p. 81) "the richer a man becomes the less is the marginal utility of money to him" (p. 81); demand schedules are presented for rich, middle class, and poor consumers (p. 88); a "pound's worth of satisfaction to an ordinary poor man is a much greater thing than a pound's worth of satisfaction to an ordinary rich man" (p. 108); "the marginal utility of a thing to anyone diminishes with every increase in the amount of it he already has" (p. 79). The importance of this inconstancy for the interpretation of the community's surplus is that the area under the market demand curve does not in general measure the welfare change for the community. Marshall explicitly and emphatically recognized this point but it has been given negligible attention, while the assumption of virtual constancy under a price change for the individual consumer has often been taken as equivalent to constancy between consumers or for any one individual at widely different income levels. On the other hand, a situation might arise in which the market demand curve and the related Marshallian surplus is acceptable for measuring welfare change. In this case consumer surplus can be used to guide decisions. But we should be careful not to treat the exception as the rule; the fact that such a case is possible does not mean that the construct is generally useful for this purpose, as Marshall recognized. Thus, if the Marshallian hallmark is interpretation of the area between p_1 and p_2 under the market demand curve as the welfare gain following a price change, we would have to conclude that Marshall was not a Marshallian.[4]

[4]Or, to quote a colleague, Tom Kompas, thinking more generally about the real Marshall and the traditionally accepted Marshall: "Someone should write a book on Marshallian economics and the economics of Marshall." It may be appropriate here to question another traditional interpretation of the work of an early writer, Jules Dupuit, whose writings have been taken as one of the first formulations of the price-marginal cost equalization rule. In an earlier article (Abouchar, 1976) we showed that for his project (a footbridge) the definition of marginal cost (long- or short-run) was particularly simple, the bridge not being part of an intercity network for which the definition of marginal cost is especially difficult, there being so many dimensions (regional distribution, weight distribution, engineering specifications—grade, turning radius, etc.—traffic volume) with respect to which costs may be marginal. Therefore, Dupuit's bridge provides no help for anything as complicated as a road network. But the point to make here, by analogy with the allegation that Marshall was not a Marshallian in his treatment of the market demand curve and consumer surplus, is that Dupuit was not a "Dupuitien" in all of his price recommendations. Thus, while he recommended that any pedestrian should be allowed to cross as long as his

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demand price exceeded the cost he would impose on society whether or not the financial recovery for the bridge should be fully compensatory, he also argued that certain fixed costs should be recovered even though they are completely unrelated to the passage of traffic and vary instead, with time (as does maintenance) or not at all (interest on capital). (Dupuit, p. 40.) It would clearly be more efficient to pay for these elements through local contributions (general taxes or a special levy), as is usually argued in the modern theory of public finance. (But not through inter-regional transfers if income distribution is not explicitly wanted for this situation.)