CIRCULATORY PHYSIOLOGY SECTION 4:
The Flow of Fluids in Tubes –
Pressure, Kinetic Energy, and Total Energy*

I. Variables of Concern in Circulations:

A. **pressure, *P*,** – force/area, usually given as mmHg or kPa

B. **volume, *V* or *q*,** given as liters or some fraction

C. **particle velocity, *v*,** given is m * s^{-1}

D. **Volume velocity**, $\dot{Q}$, given as liters/sec or minute

II. Flow in tubes

A. Pressure and volume velocity

1. Pressure is nothing more than a measure of energy per unit volume. It has units of force/area, note this is dimensionally the same as energy/ volume. Thus, when fluids move down total pressure gradients, they are also moving down an energy gradient.

2. For any ideal fluid's movements:

1. $$\dot{Q} \propto \Delta P$$

where ΔP is the pressure difference between two points between which flow occurs.
Changing this relationship to an equation, we have:

3. $$\dot{Q} = G * \Delta P$$

where *G* is the conductance. Recall that conductance is the inverse of resistance, *R*:

4. $$R = \frac{1}{G}$$

and so, by substitution:

5a,b. $$\dot{Q} = \frac{\Delta P}{R} \quad \text{-- or--} \quad R = \frac{\Delta P}{\dot{Q}}$$

Note that this equation is essentially identical to Ohm's Law.

B. Resistance and flow. Experiments by the French physician **Poiseuille** established that in smooth tubes, **equation** that:

6. $$R = \frac{8 * \eta * L}{\Pi * r^4} = 20.37 * \frac{\eta * L}{r^4} = k * \frac{\eta * L}{r^4}$$

---

kprestwich@holycross.edu

where r is the radius of the tube, η is the viscosity of the fluid (see below), L is the length of the tube, and k is a constant equal to $8 * \eta / \pi$. Thus, the longer the tube, the more resistance; the smaller the radius, the more resistance. Resistance is much more strongly dependent on radius than length.

If we substitute equation 6 into equation 5a we end up with Poiseuille's equation for laminar flow in tubes:

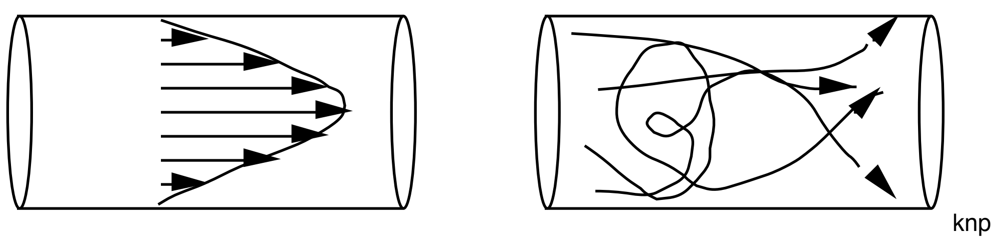
7. $$\dot{Q} = \frac{(\Delta P * \Pi * r^4)}{(8 * \eta * L)} = \frac{\Delta P * r^4}{k * \eta * L}$$

C. Pressure, Volume and Work. If pressure is energy per volume, then work is:

8. $$W = \int_{v_1}^{v_2} PdV$$

so if the pressure required to move a certain amount of blood must be increased due to an increase in resistance, the work will increase.

D. Laminar and Turbulent Flows: One important factor that determines the work of circulation and that adds some new wrinkles to our understanding of the energetics of circulation is the nature of the flow. Flow comes in two types -- laminar and turbulent as is explained in the following figure:



Profile of velocities in laminar and turbulent flow. Left: Laminar Flow -- Particles tend to flow in straight lines. Due to friciton with the walls and with other particles, flow is greatest in the center and least on the walls where the liquid encounters non-moving material in the wall. The flow velocities follow a parabolic cone with the greatest velocities in the center and no motion at all on the walls. Due to the particles sliding past each other with some small resistance they are increasingly slowed as they move towards the walls where friction is the greatest. Put another way, the friction between the walls and fluid acts as break on all the fluid but this retarding force is not entirely passed on from particle to the next the further they are from the wall. The result can be envisioned as a series of concentric cylinders -- lamina (layers) -- each with a different velocity and each sliding past each with some small dissapation of energy. Right By contrast , in turbulent flow particles move in eddys and circuitous routes and tend to impede each other far more and slide past each other far less. The result is that much more of their kinetic energy is dissipated as heat instead of motion.

(a) Turbulent flows can arise at a number of places -- for instance anywhere a vessel opens or shuts, makes a sharp bend or undergoes a large decrease in diameter. All of these factors result in changes in velocity, which we will soon see is an important determinant of flow type.

? Make a drawing of what you think the velocity profiles would look like for laminar flow in a vessel of radius = 1 and for another with radius = 3. Assume the same difference in pressure and length in each vessel. In thinking about your answer, consider what you have learned about laminar flow and be sure that your answer addresses (i) maximum (center) velocity differences that may or may not possibly exist between the two and (ii) the shape of the parabola of velocities that was explained above -- would there be a relatively broad or narrow area of peak velocities in the center of the large vessel?

(b) The important consequence of turbulent flow is that when it occurs, eqs. #5 and #7 are not valid. Instead:

9. $$\dot{Q} \propto \sqrt{\Delta P}$$

Thus for a given difference in pressure the flow is much less. Put another way, **<u>to produce a given amount of flow, the pressure difference that is required is far greater than in laminar flow.</u>** Since work done by the heart is proportional to the pressures generated to move a certain amount of blood (*dV*), the much greater work is required as higher pressures need to be produced.

(c) **<u>When will turbulent flow occur</u>?**

(i) There is a dimensionless number, called the **Reynolds Number, N_R,** which can be used to predict turbulent flow:

10. $$N_R = \frac{\rho D v}{\eta}$$

where ρ is the density (dimensionless), ***D*** is the diameter of the tube (L), ***v*** I s the average velocity of the fluid ($\frac{L}{T}$) and η is the viscosity ($\frac{T}{L^2}$).

(ii) If N_R < 2000, flow is usually laminar; between 2000 and 3000 it is transitional and if N_R > 3000 flow is usually turbulent.

(iii) Clearly **<u>factors that favor turbulent flow in the circulation</u>** include:

(a) high velocity,
(b) large diameter and
(c) low viscosity. (Density shouldn't change greatly, viscosity also shouldn't change greatly except in cases of severe anemia and polycythemia and even then other problems are certainly more significant).

? Where in the circulation is turbulent flow most likely? Why? Where is not likely? Why?

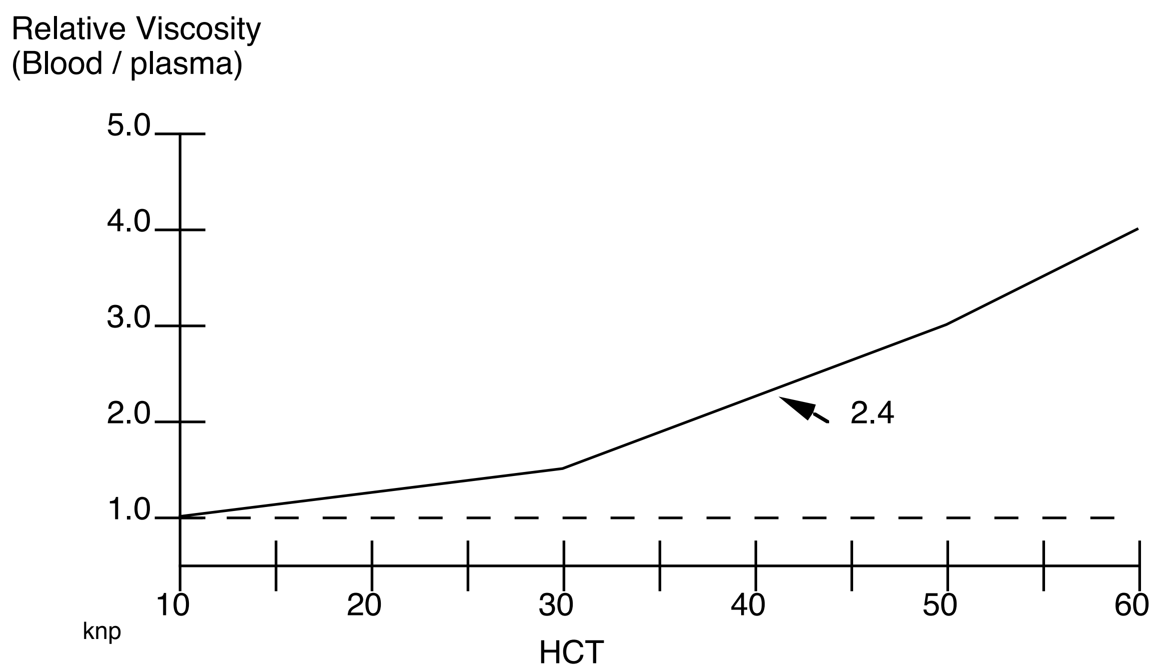
III. Viscosity:

A. <u>General</u>: **Viscosity is the within-fluid (internal) resistance to flow of fluids**. The greater the viscosity, the more friction between adjacent layers of a fluid and the slower the rate of flow for a given vessel and pressure head.

? For a certain sized vessel, what will the velocity profiles look like in a low vs. high viscosity fluid?

B. The two most important factors in determination of viscosity of the blood are (i) the hematocrit and (ii) the diameter of the vessels.

1. **<u>Effect of hematocrit on viscosity:</u>** As you might expect, the higher the hematocrit, the greater the viscosity, regardless of tube type of diameter:



This illustrates that hematocrit effects on viscosity can have important effects that easily rival normal physiological and even pathological phenomena related to vessel diameter. For instance, in severe polycythemia, the viscosity may be high enough to cause just as large of an effect on resistance as might the most common change in blood vessel diameter accompanying so-called "essential hypertension".

? Should the lowered viscosity accompanying severe anemia (which thereby reduced the work load on the heart) be viewed as a compensatory mechanism?

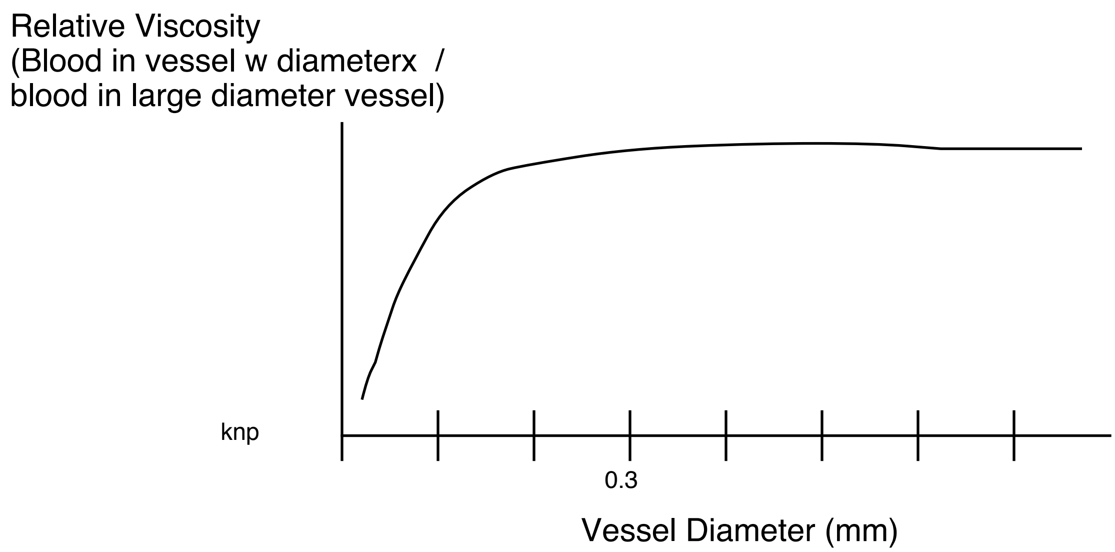
2. **<u>Effect of Blood Vessel Diameter on Viscosity</u>:**

a. <u>Blood as a Non-Newtonian Solution</u>:

i **Newtonian solutions** are those that <u>at the proper Reynolds Numbers will flow in a laminar manner</u>. Examples are fluids without relatively large particles, such as cells, and/or particles of particular shapes.

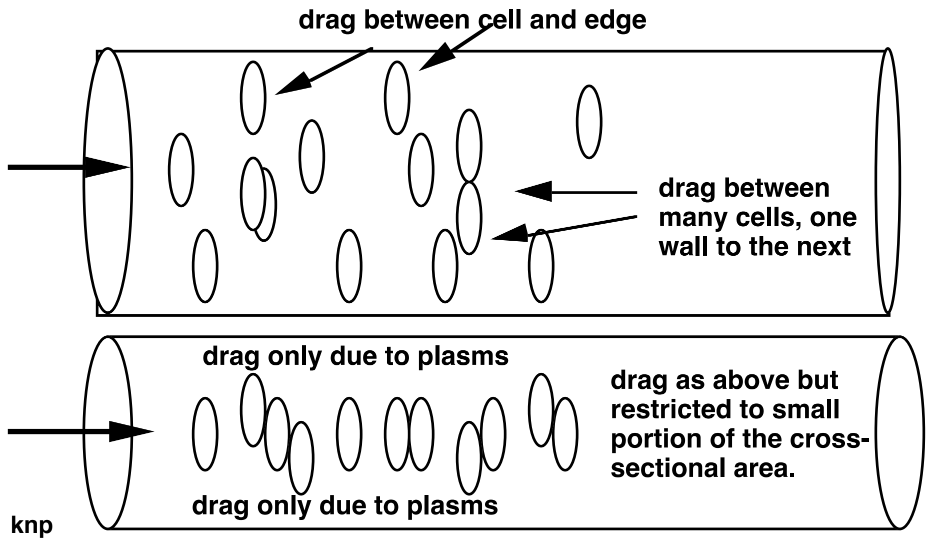
ii Blood, which is a suspension of cells in a non-cellular liquid (plasma) that contains a considerable number of colloidal particles of varying shapes (proteins) is a Non-Newtonian Fluid and its behavior is sometimes at odds with what would be predicted for Newtonian solutions. <u>One important difference between Newtonian and Non-Newtonian solutions is that v**iscosity tends not to be constant for a given set of conditions**</u>.

iii. **Fåhraeus and Lindqvist** discovered that when the diameter of a tube decreases below about 0.3 mm, the viscosity of blood tends to decrease:



iv. They pointed out that <u>since the main resistance vessels, the arterioles, are smaller in diameter than 0.3mm</u> (0.01 mm -- see earlier table), then the anomalous behavior of blood would <u>greatly reduce the resistance over what it might have been and results in lower work requirements for pumping</u> a given amount of blood. This is named the **<u>Fåhraeus-Lindqvist Effect</u>**.

v. Its cause appears to be related to the way the blood is distributed in small vessels -- cells tend to end up in the center of the blood flow and plasma near the edges, especially when the blood velocity is relatively high. The result is that in the outer areas the fluid can slide past itself without the impediment of cells and so the viscosity is essentially that of plasma whilst in the center the cells, while of higher than normal concentration, are grouped in one area and therefore as more are forced in there is no additional effect on the flow profile:



IV. Determinants of particle velocity in Newtonian fluids.

A. Particle velocity (v) and vessel cross sectional area (A). Recall that cross sectional area of a tube equals the surface area of a slice of the lumen perpendicular to the long axis of the tube; $A = 2\pi r$ where r is the radius of the tube. For rigid tubes:

11. $$v = \frac{\dot{Q}}{A}$$

Thus, when the same flow goes through different cross-sectional areas, the particle velocity varies:

12. $$\frac{v_1}{v_2} = \frac{A_2}{A_1}$$

B. This has obvious meaning when we consider the movement of blood through differently sized vessels.

V. Total, dynamic and lateral pressures of flowing liquids in tubes

A. The total pressure, $\boldsymbol{P_T}$ measured in a tube consists of the sum of two pressures, the dynamic pressure, $\boldsymbol{P_D}$, and the lateral or hydrostatic pressure, $\boldsymbol{P_H}$.

13. $$P_D = \frac{1}{2}\rho v^2$$

Notice that this equation is very much like a typical kinetic energy equation except that mass is replaced by density (mass/volume). Thus the faster a fluid moves, the greater its $\boldsymbol{P_D}$**.** This sort

of pressure makes sense to us because we experience it every time we are in motion relative to the air or water.
Hydrostatic pressure, $\boldsymbol{P}_H$, is also familiar to most of us – we notice that if we swim underwater, the deeper we go, objects such as our eardrums are distorted by the pressure (ouch):

14. $$P_H = h\rho g$$

where g is the acceleration due to gravity and h is the depth of the fluid.

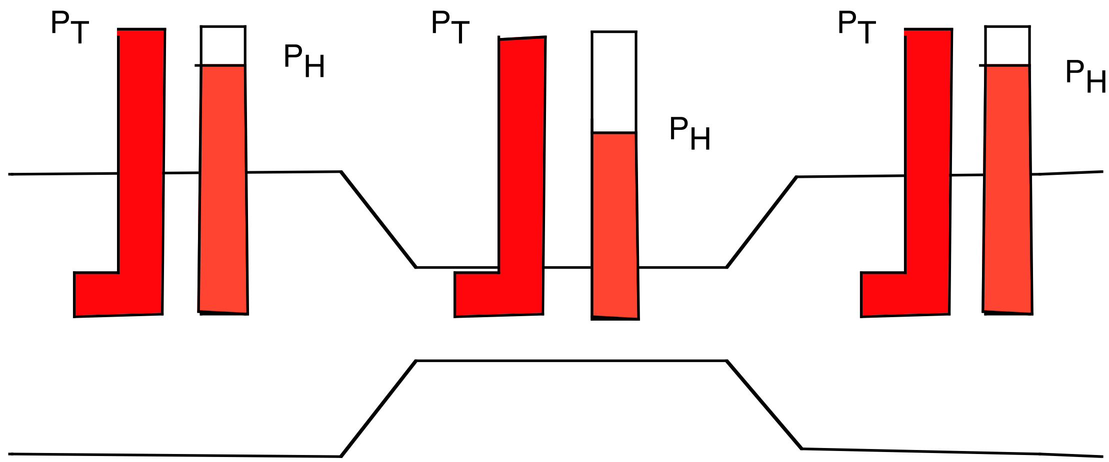
B. Imagine a situation where blood or some other fluid moves from a thick tube where the velocity is very, very low that has radius R to a thin one of r with a very high particle velocity and then back to one with radius R. Assume that only minor viscous losses occur.

1. Since R is low, then the total energy of the fluid must be regarded as roughly constant.
2. Since pressure is a measure of energy per volume, then the total energy must also be roughly constant.
3. We saw from equation 12 that the velocity in the small tube will increase by a factor of R/r.
4. If the velocity increases, then the dynamic pressure must also increase according to equation 13.
5. Since the total energy (and total pressure) must remain roughly constant (no viscous losses were assumed) then the hydrostatic pressure will drop in the high velocity low radius section of the tube.
6. When the fluid enters the next section (once again, radius R), the total energy (pressure) will remain the same and the dynamic will drop since velocity drops by r/R and the hydrostatic pressure will again increase!

C. This is easily confirmed by putting pressure transducers in the three sections of the tube.

1. One group of transducers (one to each section) has its lumen facing the fluid flow. It reads total pressure. It will read nearly the same value in each tube.
2. The second group has their lumens perpendicular to the flow direction. This reads P_H. What is observed is that the readings drop in the thin section as compared to either of the thick sections.

The diagram below illustrates an example of this. The forward facing tubes measure total pressure and the straight ones measure hydrostatic pressure only. Kinetic pressure is given below each section as a calculation.



v = 100 cm/s
$P_D = 1/2\rho v^2 = 3.8$ mm Hg
if $\rho = 1$g/ml

v = 200 cm/s
$P_D = 1/2\rho v^2 = 15$ mm Hg

v = 100 cm/s
$P_D = 1/2\rho v^2 = 3.8$ mm Hg