Appendix: Compliance and Elastance[1]

Be sure you understand this material, especially the difference between elastance and compliance. Be sure you know how to draw plots that show compliance and elastance. Also be sure you know the conventions with respect to pressure that are mentioned at the end of this section.

How do materials behave when they are stretched?

1. **STRESS**: The force acting on some object that tends to change its shape.

1. $$Stress = \frac{F}{A} = \frac{N}{m^2}$$

a. **Tensile stress** is when something acts to lengthen an object (such as a metal bar)

b. **Compressive stress** is a stress that tends to shorten an object.

c. We will consider both of these when we deal with the lungs, thorax and respiratory system in the next packet since all of these elements can be both compressed and expanded.

2. Any distortion that the material undergoes as a result of applied forces (stress) is called **STRAIN**. Thus, the strain is the relative change in a single dimension of the object in question:

2. $$Strain = \frac{(l - l_o)}{l_o}$$

where ***l*** is the length (dimension) at some time and $\boldsymbol{l_o}$ is the original resting length (dimension).

3. **ELASTIC MODULUS**: is the ratio the applied stress to the strain, i.e.,

3. $$Elastic\ Modulus = \frac{STRESS}{STRAIN}$$

For TENSILE or COMPRESSIVE STRESS TO STRAIN, the elastic modulus is called **YOUNG'S MODULUS, (Y)** and its units are:

4. $$Y = \frac{stress}{strain} = \frac{\frac{F}{a}}{\frac{(l - l_o)}{l_o}} = F * \frac{l_o}{A * (l - l_o)} = \frac{F}{A}$$

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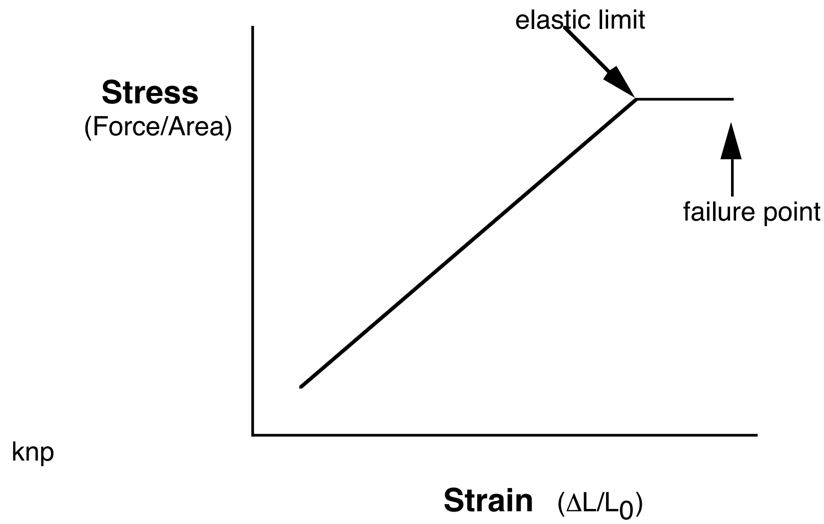
4. If we graph stress vs. strain for a simple material such as a metal wire, we will get a graph that looks like this:



The **<u>ELASTIC LIMIT</u>** is caused by the failure of the material to hold together.

5. **<u>HOOKE'S LAW</u>**: The linear relationship shown above is called Hooke's Law. The slope of this curve is Young's modulus (i.e., a tensile elastic modulus).

a. Note that the larger this modulus, the greater the force required to cause a certain % change in length.

b. In physiology, **terms such as Y that measure stiffness are referred to as measuring <u>ELASTANCE</u>**. Thus, <u>Young's modulus is really a measure or stiffness</u> since the larger its value, the stiffer the material.

c. Hooke's law also can be stated in terms of FORCE and LENGTH as:

5a. F = modulus * cross sect. area * rel. length change

i.e.,

5b. $$F = Y * A * \frac{(l - l_o)}{l_o}$$

Now, if we consider whatever we are working with as being of one unit of cross sectional area, we can ignore A, so Hooke's law becomes:

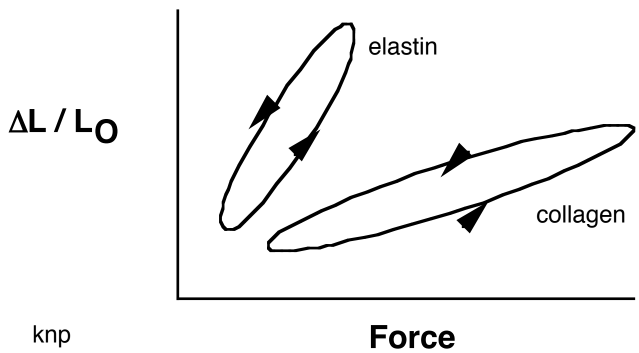
6. $$F = \frac{Y * (l - l_o)}{l_o}$$

6. **<u>COMPLIANCE</u>**: In biological systems, we often prefer to plot Hooke's law relationships differently, and put **<u>force (stress) on the x-axis</u>** and **<u>strain (change in length or volume) on the y-axis</u>**. In this case equation #6 is re-arranged to become:

7. $$\frac{l - l_o}{l_o} = \frac{F}{Y}$$ **-or-**

8. $$\text{Strain} = \frac{1}{\text{elastance}} * \text{Stress}$$

Note that the slope of the line now becomes a measure of how much something changes length with a unit force, thus it is more a measure of **stretchiness, or COMPLIANCE**. Here is a plot of the characteristics of two important substances, COLLAGEN and ELASTIN:



Note that the compliance of elastin is much greater than collagen. How about the elastance?
Also note the **HYSTERESIS**: the tendency for the material to follow different pathways during lengthening and shortening. Hysteresis is due to realignments of molecules that make up the material and therefore changing interactions between them.

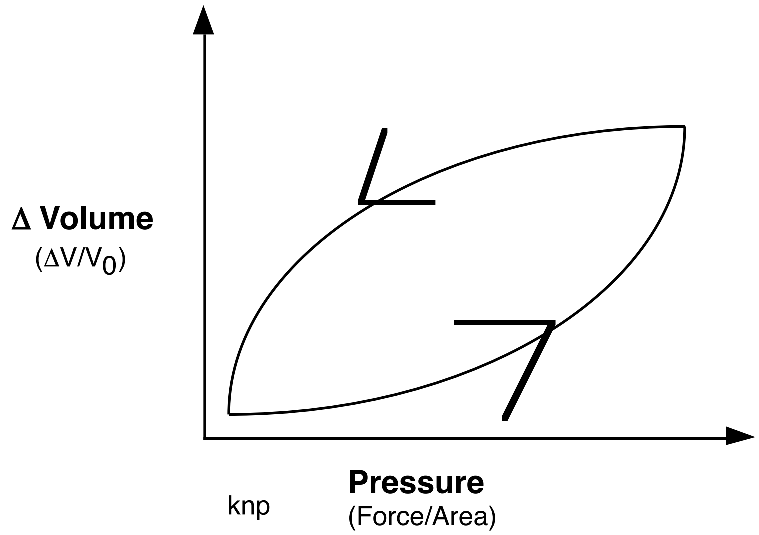
7. Lastly, note that we can plot the same sort of diagram for 3-dimensional vs. line structures. We do this by plotting the effects of **CHANGE IN VOLUME (** $\frac{\Delta V}{V_0}$ **)** vs. **PRESSURE (** $\frac{STRESS}{AREA}$ **)**. Thus,

9. $$\frac{\Delta V}{V_0} = \text{VOLUME COMPLIANCE} * \text{PRESSURE}$$

where V is the volume at some pressure, V_O is the "resting" or comparison volume and volume compliance has units of:

10. Volume Compliance = $\frac{V - V_O}{V_O} * \frac{A}{F}$ (compare with elastic compliance, eq. 6)

We can draw a graph of a **PRESSURE-VOLUME CURVE** as:



<u>IMPORTANT NOTES DEALING WITH CONVENTIONS</u>:

i) Physiologists usually report pressures in units of either **mm Hg** or **cm H_2O**.

ii) When talking about **<u>compliance and pressures used to move gases convectively</u>**, the value **of <u>0</u> pressure** will be taken to represent **<u>atmospheric pressure</u>**. "**<u>Negative" pressures</u>** are not really negative, they are instead **<u>SUB-ATMOSPHERIC.</u>**