Excitable Cell Physiology -- Problem Set #1: Does the Migration of Ions from one Compartment to Another During the Establishment of a Donnan Equilibrium Significantly Affect Ionic Concentrations?

Review the concept of capacitance before attempting the second problem. Note that problem #2 is not like anything you will see on an exam but your ability to do it will mean that you have a good understanding of the ionic basis of the resting potential in cells.

Assume that we have a very simple cell where the cation (C^+) is the only diffusible species. **Further** assume that the concentrations of C^+ are:

$[C^+]_{inside}$ = 0.1 M
$[C^+]_{outside}$ = 0.0019 M

and the temperature is 20° C.

1. Calculate the transmembrane potential for this system at equilibrium. (To make later calculations simple, round your answer to a whole number.)

Use the Nernst eq.: E_{C+} = - 58 mV* log (0.1 M/ 0.0019M)

E_{C+} = -100mV (with a bit o'rounding)

2. The capacitance of a cell membrane is always normalized to area: it is expressed as farads/area. This is obviously necessary since bigger cells with bigger membranes would have larger capacitances (since capacitance is determined by the area of the conductors and the dielectric constant). **For a typical biological membrane, the capacitance is about:**

1 mF/cm^2, i.e., 10^{-6} Farads/cm^2.

Recall that 1 Farad = $\frac{\text{coulomb}}{\text{volt}}$

(a) Calculate the amount of charge stored across one cm^2 of membrane. Give your answer in coulombs per cm^2.

Q = C*E = 1 * 10^{-6} coulombs/(V*cm^2) * 0.1 V

= 1 * 10^{-7} = 0.0000001 coulombs/cm^2

(b) For the Gibbs Donnan equilibrium you calculated above, find the number of mols of "missing" (on inside) or "excess" (on outside) C^+ per cm^2 of membrane. In other words, how many positive ions are out of position to establish the voltage seen at equilibrium for the situation described in the earlier problems? An alternative way to put this is that you are going to find the number of excess A- on the inside or C+ on the outside.
You will need the following constant:

1 Faraday of charge = 1 mol of univalent charged particles
= 96,500 coulombs/mol

mols of excess (or missing) C^+/ cm^2
= number of mols of "unbalanced" charges
= (mols of charges in on cm^2 of membrane) / (charges/mol)

substituting the # of coulombs of charge from part (a)…

= $(1*10^{-7}$ coulombs/cm^2) / ($9.65*10^4$ couls/mol chrg)

= $1.04*10^{-12}$ mols of charge /cm^2

(c) Use your answer in (b) to calculate the total number of excess – or missing + charges (univalent ions) on the inside (and vice versa on the outside) per cm^2.

All we need to do is multiply mols of charge per unit membrane times Avogadro's number
= ($1.04*10^{-12}$ mols of charge /cm^2) * ($6.02*10^{23}$ particles/mol)

= $6.24*10^{11}$ charged particles /cm

This may seem like a lot of particles to have moved across the membrane (given what I told you in class) but lets put it in context:

(d) Now, assume that for this typical axon, we are concerned with a:
1 cm($1* 10^{-2}$ m) long section with a
diameter of 10 μ ($1 * 10^{-5}$ meters).

Therefore, in this section, the axon has the following **total membrane surface area**:

= length * area = L * diameter * pi = $1*10^{-2}$ m * pi * $1*10^{-5}$ m

$= 3.142*10^{-7}\ m^2 = 3.142 * 10^{-3}\ cm^2$

What is the **number** of charged particles stored in a membrane capacitor of this surface area?

Here we use the value from (c) which gives us the number of separated charged particles per area at -100 mV and multiply this by the axon's actual surface area. Thus:

= actual area (cm^2) * charges per cm^2 @ -100 mV

= ($3.142 * 10^{-3}\ cm^2$) * ($6.24*10^{11}$ charged particles /cm)

= $1.96 * 10^9$ charged particles

(e) How many charged particles are present in the cytosol of this same bit of axon (1 cm with a 10 micron diameter)?
The internal concentration of C^+ was given at the start as 0.1 M. We can calculate the volume as:

= cross sectional area * length = $(\text{diameter}/2)^2$ * pi * L

$= (5*10^{-5}\ m)^2 * pi * 1*10^{-2}\ m = 7.85 * 10^{-11}\ m^3$

and since $1\ cc = 1\ ml = 10^{-6}\ m^3$ then the volume is, in ml:

= **$7.85 * 10^{-5}$ ml**

So, how many C^+ particles reside in this tiny volume if the concentration of C^+ is 0.1 M?

First let's figure out how many mols are there. The concentration of C+ is 0.1M or 0.1 mols C+ / L. Since there are 10^3 ml in one liter then:

Mols C+ / ml = (0.1 mols / L) / (10^3 mL/L) = $1 * 10^{-4}$ mols / ml

What is the total number of particles present in this 1 cm long by 10 micron diameter (10^{-5} m) section of axon?

 # = (Total # mols C^+) * (# particles / #mols)
expanded:

#= (mols C^+ / ml) * (# of ml) * (# particles / mol)
where the first two parts of the equation were calculated and given in the setup for this problem and the other is simply Avogadro's number:

= (1 * 10^{-4} mols/ml) * (7.85 * 10^{-5} ml) * 6.02 * 10^{23} ions/mol
= 4.73 * 10^{15} C^+ in the 1 cm section of cell

(f) What is the ratio of the number of charges separated in the establishment of the Gibbs-Donnan Equilibrium to the # number of ions still inside the cell?

Using the answers from (d) -- **1.96 * 10^9 charged particles)** and (e) -- **4.73 * 10^{15} C+** in the cell volume of this 1 micron section, we find:

% of total + charges that had to separate 1.96 * 10^9 out of balance charged particles / 4.73 * 10^{15} C^+ inside *100

= 4.15 *10^{-5} % = 0.0000415%

(g) In the establishment of a Gibbs Donnan equilibrium, is there any meaningful change in the concentrations of the ions inside the cell?

Obviously not! See (f)