The following data were collected by measuring rates of metabolism in an endothermic homeotherm over a range of ambient temperatures.

Ambient Temperature ^oC	Rate of Metabolism cc O_2 / (g*h)
0	1.482
10	1.092
within zone of thermal neutrality	0.702

a. Estimate the minimal thermal conductance, C_{min}.

We are told that the metabolic rate within the Thermal Neutral Zone (TNZ) is 0.702 ml O_2 / (g*h); we are also given two other values of $\dot{V}_{O_2}$ that show an increase with a decrease in ambient temperature. This, along with the fact that both of these values are greater than $\dot{V}_{O_2}$ within the TNZ shows that they represent $\dot{V}_{O_2}$ values below the TNZ. We know that C_{min} occurs below the TNZ and it is equal to the absolute value of the slope of the $\dot{V}_{O_2}$ vs. T_a curve. If you are unsure of this, recall that if $\dot{V}_{O_2}$ has units of ml O_2 / (g*h), then C_{min} must have values of ml O_2 / (g*h*^oC) -- check this out dimensionally with Newton's Law of Cooling. So:

$C_{min} = \Delta \dot{V}_{O_2}$ / Δ T_a -- for values below the lower limit of thermal neutrality (T_{ll})

So:

C_{min} = ABS[(1.482 - 1.092) ml O_2 / (g*h) / (0 - 10)^oC]

C_{min} = ABS[0.39 ml O_2 / (g*h) / -10 ^oC]

C_{min} = ABS[-0.039 ml O_2 / (g*h°C)]

C_{min} = 0.039 ml O_2 / (g*h°C)

b. Estimate core body temperature. Do this both with a graph and by solving an equation directly.

Let's solve the equation directly. We need to re-arrange it to find T_B:

$$\dot{V}_{O_2} = C(T_B - T_A)$$

$$T_B = T_A + \dot{V}_{O_2} / C$$

We can use the $\dot{V}_{O_2}$ for any point the T_{LL} or below provided we know T_a. For our example, we only know T_a for $0°$ and $10°$ C; we can use either of these and their $\dot{V}_{O_2}$ value along with C_{min} since either is below the T_{LL}. Let's use $0°$ C. We substitute:

$$T_B = T_A + \dot{V}_{O_2} / C = 0° C + \{1.482 \text{ ml } O_2 / (g*h) /\ 0.039 \text{ ml } O_2 / (g*h°C)\}$$
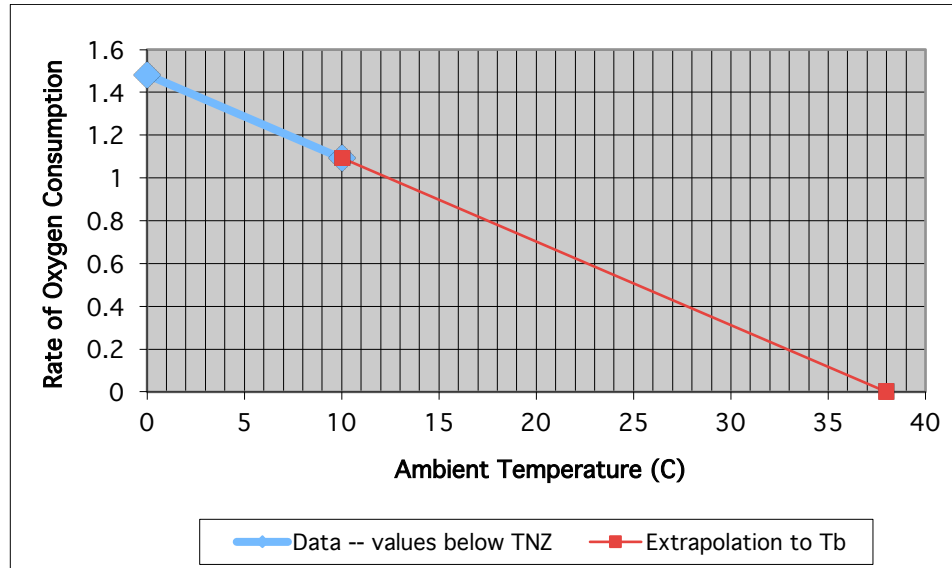
$T_B = 0° C + 38° C = 38° C$

Just to show this works for the $10°$ C point, here it is again:

$T_B = T_A + \dot{V}_{O_2} / C = 10 + 1.092/0.039 = 10 + 28 = 38$

The graph uses the same technique. All we do is to extend the line for $\dot{V}_{O_2}$ vs. T_a to the case where $\dot{V}_{O_2}$ = 0. That is the T_B. Here is the logic:

$$T_B = T_A + \dot{V}_{O_2} / C$$

if $\dot{V}_{O_2}$ = 0 then $\dot{V}_{O_2}$ / C also equals zero and $T_B = T_A$. The graph is on the top of the next page:

I recommend that you solve it algebraically as it is more accurate. But the graph is a nice visual short cut.

c. Determine the lower limit of the zone of thermal neutrality.

Once again, this is best done by solving the equation. In this case we want to find the T_a at the lower limit of thermal neutrality; *i.e.,* T_{LL} . Recall that at T_{LL} that these things are all true:

- **Conductance is C_{min} (0.039 ml O_2 /(gh°C) in our example, see first question)**
- **Metabolism is basal (*i.e.,* 0.702 ml O_2 /(gh) in our example)**
- **Normal T_b -- we're euthermic! -- 38° C in our example**

 Starting as usual with Newton's Law of Cooling:
$$\dot{V}_{O_2} = C(T_B - T_A) \text{ or}$$
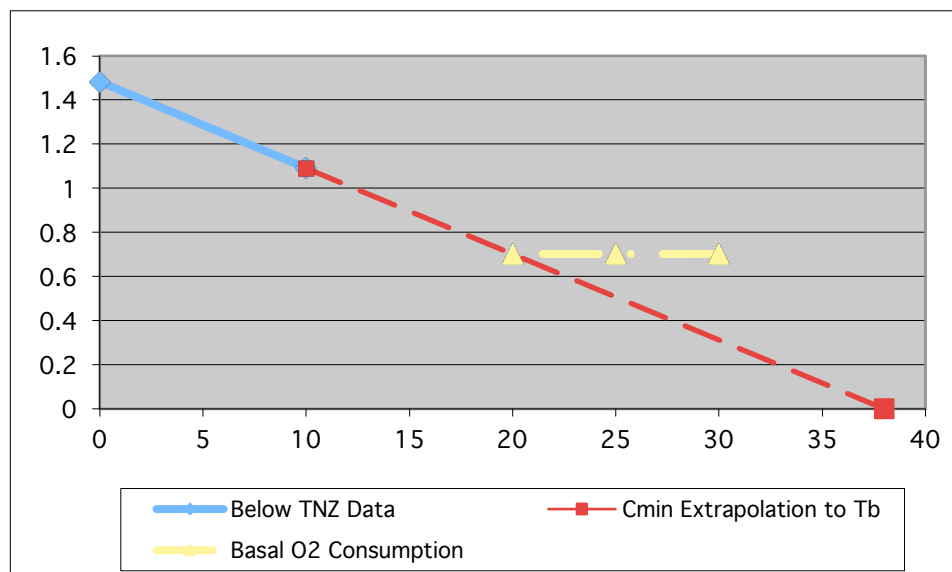$$\dot{V}_{O_2 \text{ basal}} = C_{min} (T_B - T_{LL})$$

and re-arranging:

$$T_{LL} = T_B - \{\dot{V}_{O_2\ basal} / C_{min}\}$$

$$= 38^\circ\ C - \{0.702\ ml\ O_2 /(gh) / 0.039\ ml\ O_2 /(gh^\circ C)\}$$

==$= 38^\circ\ C - 18^\circ\ C = 20^\circ\ C$==

Here it is graphically -- all I did was extend the basal metabolism line (see given data) to where it intersected the C_{min} line. The intersection point is T_{LL}:

d. What would be the new lower limit of the zone of thermal neutrality if insulation is decreased to one-half its original value?

Once again, the best way to find this is to calculate it. If the insulation is cut (literally perhaps) to half its previous value, the C_{min} will double since I = 1/C. If we use the new value of C_{min} with the previously determined T_B and basal $\dot{V}_{O_2}$ we can find the new T_{LL} using the same equation as in the last problem:
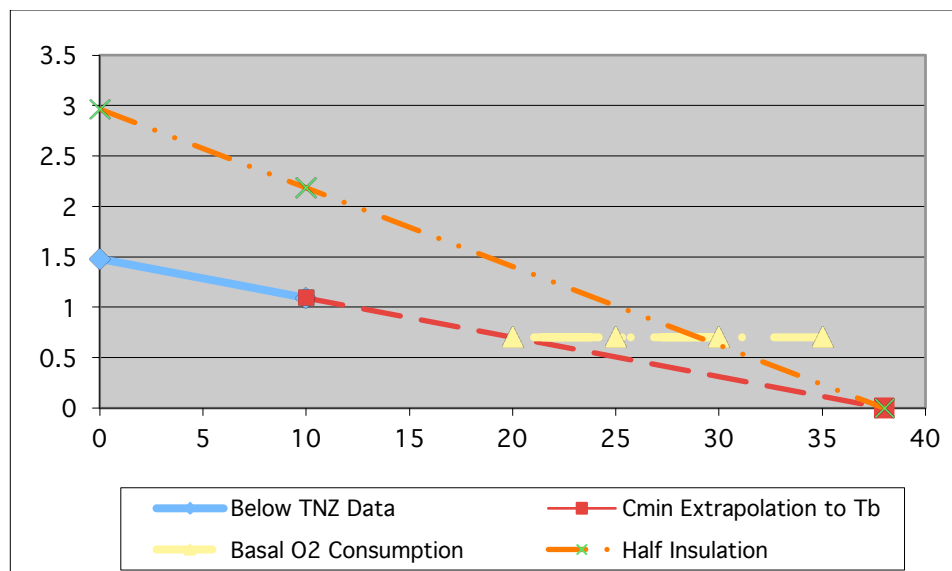
$$T_{LL} = T_B - \{ \dot{V}_{O_2 \text{ basal}} / C_{min} \}$$

$$T_{LL} = 38\ {}^{o}C -$$
$$\{0.702\ \text{ml } O_2 / (gh) / 2 * (0.039\ \text{ml } O_2 / (gh{}^{o}C))\}$$

$$= 38\ {}^{o}C - (0.702 / 0.078)\ {}^{o}C = 38 - 9 = \boxed{29\ {}^{o}C}$$

Thus, T_{LL} increased 9 ${}^{o}C$; one would expect an increase in T_{LL} if insulation decreased. This would happen in warm weather acclimation.

Here is the same thing graphically -- all that I have done is added a second line with a greater minimal thermal conductance:

Note that I have also extended the TNZ to the right (higher temperatures) as would typically be the case. However, I have no way of knowing where the upper limit of the TNZ is found (T_{UL})