Estimating key macroeconomic relationships at the undergraduate level: Taylor rule and Okun's Law examples

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Abstract

This paper presents some Excel-based exercises that allow students to estimate some key macroeconomic relationships: Okun's Law and the Taylor rule. The Okun's law exercise has the additional benefit of providing estimates for long-run GDP growth. The Taylor rule exercises give students the opportunity to replicate, and then improve upon a seminal paper in macroeconomics. Overall, these exercises give students an introduction to some key aspects of conducting empirical research in macroeconomics, including manipulating models into a form that can be estimated and gathering and manipulating data. In addition, the exercises provide students with useful spreadsheet skills that can be used in other assignments and other arenas, long after graduation.

Supplementary web page for paper

http://www.holycross.edu/departments/economics/mcahill/AEA2006/

Introduction

Students sometimes find it difficult to connect complex macroeconomic theory to real-world observation. One problem is that theoretical models of macroeconomic relationships are often complex, as many models have several equations. Further, the econometric techniques necessary for successfully estimating macroeconomic relationships are far beyond the grasp of most undergraduate students. However, there are a few macroeconomic models that can be simply estimated at the intermediate level. This paper suggests two such relationships: Okun's Law, and the Taylor rule.

In addition to being practical empirical macroeconomic exercises, these examples can provide a gateway for discussion on a number of central topics in macroeconomics. They also provide practice using some important research methods. This paper provides a detailed description of carrying out these exercises in an undergraduate class. Laboratory instruction sheets and completed Excel spreadsheets can be downloaded from http://www.holycross.edu/departments/economics/mcahill/AEA2006/.

Estimating Okun's Law

The Okun's Law relationship (see Okun (1962)) is well-known in economics. It posits a stable relationship between the GDP gap and the unemployment rate (relative to the natural rate of unemployment). Okun's Law is often written in the following form:

$$\omega(U^{*}-U) = (Y - Y^{*})/Y^{*} \qquad (1)$$

where U is the unemployment rate, Y is real GDP, and an asterisk represents potential or natural rate levels of variables. Equation (1) states that for every 1% point the unemployment is below the natural rate, GDP is ω% above potential GDP (and vice versa). Usually, the value of ω, the "Okun's Law parameter," is suggested to be about 2, suggesting that GDP falls two percent relative to potential when the unemployment rate rises by one percentage point. Unfortunately, U^{*} and Y^{*} are difficult to estimate, making it impossible to estimate ω. However, data are available to estimate the "growth rate form" of Okun's Law. After some manipulation of equation (1) (and a little bit of hand-waving), the "growth rate version" of Okun's Law restates the relationship as:

$$dY/Y = -\omega dU + dY^{*}/Y^{*} \qquad (2)$$

In words, this equation states that the real GDP growth rate is equal to the potential GDP growth rate less the product of Okun's law coefficient and the change in unemployment rate. (See *e.g.* Mankiw (2003) for a description of the growth rate version of Okun's Law. See Appendix A for a derivation.) This relationship is easily estimated using an ordinary least squares (OLS) regression on real GDP and unemployment data, as is depicted in texts such as Mankiw's. While a simple OLS investigation ignores some potentially important econometric issues, a nice benefit to this exercise is that it not only provides a reasonable estimate of Okun's Law parameter, but also of the growth rate of potential GDP. By conducting the exercise in different time periods, students obtain remarkably stable estimates for Okun's Law parameter and changing estimates of potential GDP growth which closely correspond to results reported in the literature. I use this exercise in my intermediate-level Macroeconomics class. Lab directions are found in Appendix B.

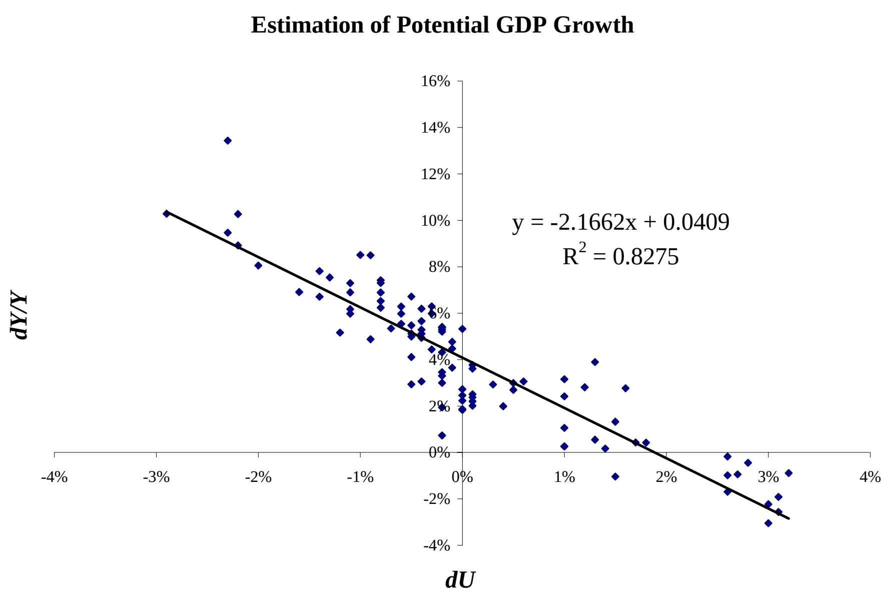
Data to estimate equation (2) from 1949 to the present are available from the *FRED II* database from the Federal Reserve Bank of St. Louis (2005). Unemployment is available at the monthly frequency, and real (chain-weight) GDP at quarterly intervals. Obviously, some manipulation must be done to make the two series compatible. After some experimentation, I have found that it is best to use the unemployment rate for the last month of each quarter, and compute changes in unemployment and GDP from the previous year. While it may be a valuable exercise to show students how to gather and manipulate the raw data into its usable form, I usually provide the students with a converted data sheet. In addition to making the appropriate calculations, I divide the data up into three time periods, starting in 1949, 1973, and 1997, corresponding to changes in the growth rate of potential GDP. I also make sure that the change in unemployment data appears in the first data column. A portion of the a sample sheet can be found below in Figure 1:

Figure 1: Partial sample data sheet

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Year**	**quarter**	***dU***	***dY/Y***		**Year**	**quarter**	***dU***	***dY/Y***		**Year**	**quarter**	***dU***	***dY/Y***
2	1949	1	1.00%	1.06%		1973	1	-0.90%	7.69%		1997	1	-0.30%	4.49%
3	1949	2	2.60%	-0.99%		1973	2	-0.80%	6.43%		1997	2	-0.30%	4.37%
4	1949	3	2.80%	-0.45%		1973	3	-0.70%	4.86%		1997	3	-0.30%	4.79%
5	1949	4	2.60%	-1.69%		1973	4	-0.30%	4.16%		1997	4	-0.70%	4.34%
6	1950	1	1.30%	3.89%		1974	1	0.20%	0.70%		1998	1	-0.50%	4.69%
7	1950	2	-0.80%	7.30%		1974	2	0.50%	-0.17%		1998	2	-0.50%	3.80%
8	1950	3	-2.20%	10.27%		1974	3	1.10%	-0.60%		1998	3	-0.30%	3.71%
9	1950	4	-2.30%	13.44%		1974	4	2.30%	-1.93%		1998	4	-0.30%	4.51%
10	1951	1	-2.90%	10.28%		1975	1	3.50%	-2.26%		1999	1	-0.50%	4.24%
11	1951	2	-2.20%	8.92%		1975	2	3.40%	-1.82%		1999	2	-0.20%	4.42%
12	1951	3	-1.10%	6.90%		1975	3	2.50%	0.82%		1999	3	-0.40%	4.43%
13	1951	4	-1.20%	5.17%		1975	4	1.00%	2.54%		1999	4	-0.40%	4.70%

Microsoft Excel allows a two-variable regression equation to be estimated within a scatter-plot chart. The chart itself gives students a visual depiction of the link between two variables, and an idea of the tightness of the relationship. Thus, the first step to estimate Okun's Law in Excel is to plot the data (first for the 1949-1972 period) in a standard scatter plot. To estimate the regression equation, click on the "Chart" menu, then on "Add Trendline". Choose the "Linear" type; then click on the "Options" tab, and select the "Display equation name" (and "R-squared value" if desired). Click on OK. The resulting equation is in $y = mx + b$ form. For the 1949-1972 period, the chart looks like the following:

Figure 2



Students then must interpret the equation as the growth rate version of Okun's Law. In this time period, $\omega = 2.2$, and $dY^*/Y^* = 4.09\%$. These correspond very closely to other estimates; for example, trend real GDP growth in this period is 3.89%. When the exercise is repeated for the other time periods (1973-1996 and 1997-2004), estimates of ω are 1.83 and 1.95 respectively, and for potential GDP growth 3.01% and 3.40%.

The results suggest that the Okun's Law parameter is relatively stable, at a value of about 2, but GDP growth has undergone some significant changes over time. As such, this lab can provide a gateway for discussion of a number of issues, including the slowdown in GDP growth, the new economy hypothesis, the stability of the natural rate of unemployment, and the meaning of "laws" in economic theory. In a more practical sense, by showing students this relatively stable relationship between unemployment and GDP growth, the link between these two facets of the economy can be made more concrete when short run models of the economy are developed in class. Perhaps most importantly, students are empowered by estimating a key relationship and have gained some useful spreadsheet skills that can be used in other assignments.

The Taylor rule

The interest rate targeting scheme that has become known as the Taylor rule was first presented in Taylor (1993) as an example to show how monetary rules can be helpful in formulating policy, but at the same should not be followed slavishly. In fact, a central part of the paper discusses how the Fed rightly deviated from the rule’s prescriptions under special circumstances in 1990 – the spike in oil prices at the start of the 1990 Iraq war and the reunification of East and West Germany. However, the fact that the rule seemed generally to follow the Greenspan administration’s actual federal funds rate target so closely led to considerable attention in the popular press and a large academic literature for a time. More recently however, Orphanides (1997) showed that the rule is not likely to be useful using real-time data.

Despite its flaws as a practical tool, the Taylor rule has sparked a renewed interest in monetary rules, and is covered in many intermediate-level macroeconomics texts, including Mankiw (2003), Hall and Papell (2005) and others. Furthermore, it is very useful for evaluating historical monetary policy regimes. For example, Judd and Rudebusch (1998) estimate values of the key parameters using data for three recent Federal Reserve governorships. This paper finds results that mainly support preconceptions about the policy regimes, but offer some surprises as well.

One of the attractive qualities of the Taylor rule is that, like Okun’s Law, it is a simple equation that seeks to explain a key macroeconomic relationship. The rule can be written as:

$$i_t^T = r^* + \pi_t + \alpha_1 \left(\pi_t - \pi^*\right) + \alpha_2 \left(Y_t - Y_t^*\right)/Y_t^* \qquad (3)$$

where i is the nominal federal funds rate, T denotes target, r^* is the long-run real federal funds rate, π is the inflation rate, π^* is the inflation rate target, Y is GDP, Y^* is potential GDP, and α_1 and α_2 are the policy reaction parameters. Taylor (1993) originally suggested $r^* = \pi^* = 2\%$ and $\alpha_1 = \alpha_2 = 0.5$, almost on an *ad hoc* basis for illustrative purposes.

In class, I have found it useful to replicate some of the Taylor rule research to illustrate a number of points in a research methods seminar. I also plan to use these exercises in a *Monetary Theory* course to be offered in the near future. In one project, students replicate Taylor's (1993) simulation of the rule in Excel, and then use statistical analysis (rather than Taylor's (1993) rough "eyeballing" analysis) to judge how close the rule's interest rate target is to the actual federal funds rate. Students then use Solver to find values for α_1 and α_2 that maximize the correlation, and find they are different from Taylor's (1993) suggestion. In the second project, students replicate a simplified version of the Judd and Rudebusch (1998) regression equations to get a better estimate of α_1 and α_2 then by the correlation analysis. While this approach again ignores some technical econometric issues, students are forced to do some careful manipulations and in the end are able to get interesting results similar to those found in the literature.

Replicating and improving on Taylor (1993)

While there has been considerable detailed and nuanced economic research into the Taylor rule since the original paper, the simplicity of Taylor (1993) paper provides a vehicle for students to replicate and improve upon seminal research. Because the rule was originally used for illustrative purposes only, the extent of the analysis of the Taylor rule coefficients in Taylor (1993) is "eyeballing" a time series plot of the rule-suggested federal funds rate and the actual federal funds rate. The exercise described below closely replicates the Taylor (1993) exercise and then improves on it by expanding the time period covered, statistically testing how close the Taylor rule is to the actual federal funds rate, and then using an Excel command to find values for α_1 and α_2 that yield even closer correlation. Appendix C presents a sample assignment for this lab, in the form of writing up results as a paper.

The first step is to define the time period of the exercise. While Taylor (1993) examined only the post-1987 Greenspan period, it is useful to look back to 1970. This allows an analysis of three Federal Reserve Board chairmanships: Arthur Burns (2/1/70-1/31/78), Paul Volcker (8/6/79-8/11/87), and Alan Greenspan (8/11/87-present).[1] The second step is to gather the appropriate data for the federal funds rate, inflation rate, GDP, and potential GDP from *FRED II* (Federal Reserve Bank of St. Louis 2005). Through experimentation, I have found that the monthly effective federal funds rate, monthly CPI-U,

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[1] See Federal Reserve Board of Governors (2005) for dates. Note that this exercise ignores the G. William Miller administration (3/8/78-8/6/79).

quarterly real chain-weight GDP, and quarterly potential GDP series work best (starting in 1969). In addition, it is best to save each to its own worksheet.[2] These choices differ a bit from those employed by Taylor (1993), partly because some of these data series were not available when the paper was written.

The next step is to convert the data into the form needed to simulate the paper. This requires some careful choices. Starting with the federal funds rate, what seems to work best is to average the federal funds rate over the three months of each quarter (starting with 1970-03-01) by creating a new column in the data sheet with the appropriate formula in the last month of each quarter. Leave the other months blank.[3] The data should also be converted to decimal form by dividing by 100. Turning to the inflation rate, what seems to be most effective is to find the percent change in the CPI from the previous year for the last month in each quarter. To accomplish this, create a column in the CPI data sheet to make this calculation, again leaving all other months blank. Next, the estimate of the percent GDP gap is needed. To calculate this, first copy the GDP potential data to the real GDP sheet, being careful to line up time periods carefully. Then, create a new column to calculate the percent GDP gap.

Now, combine all the series into a single worksheet, starting with the first quarter of 1970. The first row should contain descriptors and the first column the date. I have found that the plots are cleaner if a decimal date column is created, where 1970.00 is the first quarter of 1970, 1970.25 is the second quarter, etc. This is easily done copying a formula or using the Edit/Fill/Series/Linear command.[4] The next column should contain the federal funds rate data. To copy only the cells with entries (at the quarterly intervals), use the “Autofilter” feature: click on the column with the data, then select Data/Filter/Autofilter. Now, a drop-down menu appears under the data column; select “(nonblanks)”, and only those cells will be displayed. Copying the column to the combined data sheet will copy only the visible cells.[5] Repeat this process for the other data series.[6]

The final step in preparing the spreadsheet is to insert four blank rows at the top of the sheet to contain the parameter values for r^*, π^*, α_1 and α_2. As will be clear below, it is best to place the values in

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[2] It is simple enough to combine separate files into the same workbook file using the Edit/Move or Copy Sheet… command. The dialog window allows a sheet to be moved or copied to a different workbook.
[3] It is still straightforward to copy the formula down the column by selecting the two blank cells and the formula cell and copying down).
[4] In this notation, the first term refers to the menu, and the terms following a front-slash refer to menu choices.
[5] Note: When pasting onto the combined data sheet, make sure to Edit/Paste Special/Values.
[6] It is also advisable to format the cells so the data is easily viewed; e.g. use two-decimal percentage format for the data.

the same column as the new “TR rule target” value will be calculated. The finished data sheet should look something like the one depicted in Figure 3 below.

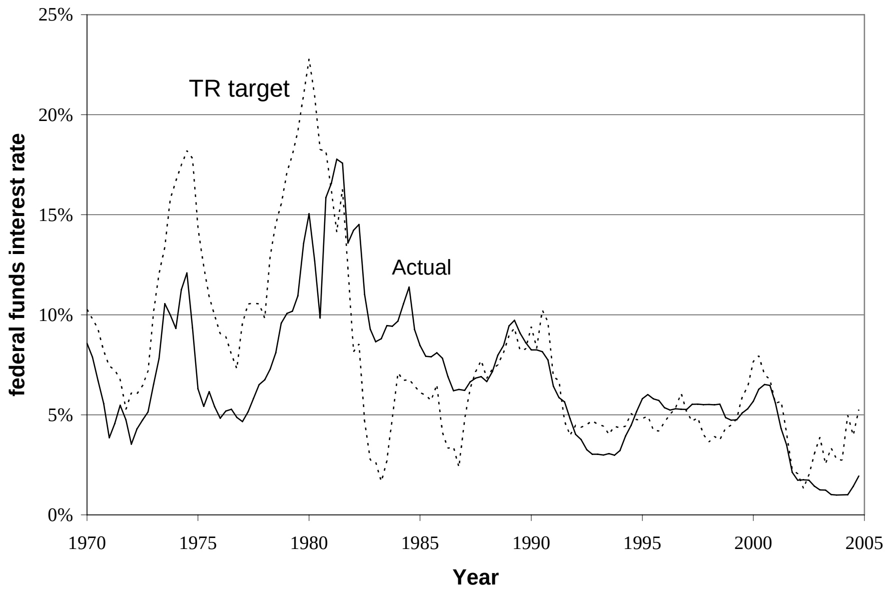
Figure 3: Taylor rule set-up

	A	B	C	D	E
1				r*	0.02
2				pi*	0.02
3				alpha1	0.5
4				alpha2	0.5
5	Year	fed funds c	inflation rat	GDP gap	TR rule tar
6	1970	8.57%	6.09%	0.25%	
7	1970.25	7.88%	6.01%	-0.44%	
8	1970.5	6.70%	5.66%	-0.42%	
9	1970.75	5.57%	5.57%	-2.31%	
10	1971	3.86%	4.44%	-0.43%	
11	1971.25	4.56%	4.38%	-0.69%	
12	1971.5	5.47%	4.08%	-0.72%	
13	1971.75	4.75%	3.27%	-1.24%	
14	1972	3.54%	3.50%	-0.30%	
15	1972.25	4.30%	2.96%	1.22%	
16	1972.5	4.74%	3.19%	1.34%	
17	1972.75	5.14%	3.41%	2.13%	

Now the sheet is ready to calculate the Taylor rule target. In the next column (Column E in Figure 3), write a formula to calculate the Taylor rule target value using the data column and parameter value cell references as appropriate. Because we are later going to copy this column, the formula should be written with mixed-references: for the data, hold the column name fixed (with a $), and for the parameter values, hold the row name fixed. So, the formula in E6 on Figure 3 above should be: =F$1+$C6+F$3*($C6-F$2)+F$4*$D6. Once this is entered, copy the formula down through the end of the time periods.

Now we are ready to compare the Taylor rule target to the actual federal funds rate. Following Taylor (1993), the first step is to plot the series together. Selecting the date column data, federal funds rate data, and TR target data (holding down the Ctrl key allows for disjointed sections to be selected together), and create a scatter plot. The result should look like the plot in Figure 4:

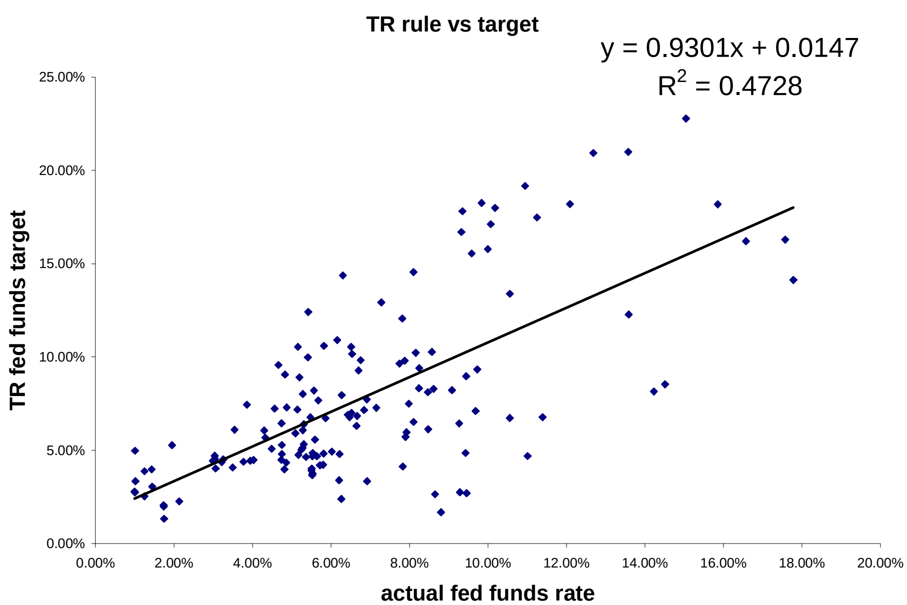
Figure 4: Taylor rule plot



And analysis of the plot can lead to some interesting discussions about the history of Federal Reserve policy. First focusing on the Greenspan years (1987-present), it does look as though the Taylor rule explains federal funds rate target choices fairly well, though is more smoothed, and with a couple of notable deviations: in 1993-1994 (when the Fed was reacting to the slow recovery after the 1990-1991 recession), 1997 (when the Fed was reacting to the Asian financial crisis), and in the most recent recovery (when the Fed has been much slower to raise its target than the Taylor rule prescribes). Interestingly, while some criticized the Fed for reacting too slowly to the 1990-1991 recession, the actual federal funds rate almost exactly corresponds to Taylor rule target prescriptions. More interesting is the previous Volcker administration (8/79-8/87), when the Federal Reserve generally held the federal funds rate target much higher than the rule suggests (to combat inflation), and the Burns period (1970-1978) when the actual federal funds rate was held much below the rule-suggested rate (either accommodating inflation or fighting a perceived recession).

Of course, examining the plot is an unsophisticated way to draw conclusions. A slightly better method is to examine correlation coefficients and simple regression lines. Such analyses can be conducted for the full period, or for separate Fed governorships. A regression line analysis similar to that used in the Okun's Law exercise is a good first step in this analysis. Such a plot of the full period appears below in Figure 5:

Figure 5: Taylor rule Scatter Plot Analysis



Another option is to compute and test correlation coefficients. Correlation coefficients are calculated in Excel with the =CORREL command; a row is easily added at the end of the spreadsheet to accomplish this. I put the formula at the end of the data in row E, and fixed the column reference for the actual fed funds data. For the Greenspan period, the correlation coefficient is 0.84. As a further exercise, the statistical significance of this value can be tested in Excel.[7]

The next part of the exercise is to explore whether the assumed values for α_1 and α_2 are optimal. For this exercise, copy the entire column of TR target data (column E) to the next column (F).[8] Verify that the correlation formula at the end of the data in this column to calculates the correlation coefficient

[7] To accomplish this, create a row for the *t*-test. The first column should contain the number of observations (using the =COUNT command); in the next column, the test statistic is calculated (for correlation, the test statistic is $t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ with *n*-2 degrees of freedom see e.g. Mason (1986) or any elementary statistics text). The *p*-value is calculated with the =TDIST(*t*, *df*, *tails*) command. For the Greenspan period, the correlation coefficient is statistically different from 0 and 1 at the 99% level of significance.

[8] If the formula was set up correctly, the values will not change.

between this column and the actual federal funds rate data column. Now, we use the Solver Add-in to find the values for α_1 and α_2 that maximize the correlation coefficient.[9] Select the cell with the correlation calculation (set, say to the Greenspan period), and then open the Tools/Solver box. Select the correlation coefficient cell as the "Set Target Cell", and the "By Changing" cells should be set to the α_1 and α_2 addresses (in the example here, F3:4). After pressing "Solve," and "Keep Solver Solution," Excel will have changed the values in the α_1 and α_2 cells. The values are considerably different than the Taylor assumptions – 0.39 and 0.87 respectively – but the correlation coefficient is only slightly higher (0.87). This suggests that the Taylor rule target is not very sensitive to particular parameter values. Adding this new series to the time series plot will informally verify this.

Estimating Taylor rule parameters

The next laboratory assignment uses the regression features of the Excel Analysis ToolPak to estimate values for Taylor rule parameters. (See Appendix D for a sample lab sheet.) While the limited econometric capabilities of Excel leave open a number of critical empirical issues, some interesting results can still be obtained. This exercise loosely follows the Judd and Rudebusch (1998) article which conducted a more sophisticated version of this exercise. The purpose of Judd and Rudebusch (1998) was to estimate parameter values for different Federal Reserve Board governorships to better understand the goals of each administration. Results suggested that the Taylor rule fits the Greenspan administration the best (albeit with an α_2=0.9), and the Greenspan administration used reasonable values for π^* and r^* of about 2-2.5%.[10] The model fit is worst in the Burns period, where r^* appears to be too low and α_1=0; this meets with popular sentiment that the Federal Reserve during that period did not have any inflation target

[9] If Solver is not installed, click Tools/Add ins and check "Solver."
[10] Judd and Rudebusch (1998) and Taylor (1993) suggest this is a reasonable long-run value for the real federal funds rate.

and over-stimulated the economy.[11] During the Volcker administration, r^* is estimated to be too high, likely reflecting the disinflationary policy.

Before conducting the exercise, it is worth noting that it is not possible to independently estimate the two constant terms r^* and π^* simultaneously. Instead, the following equation can be estimated:

$$i_i = \beta_0 + \beta_1 \pi_t + \beta_2 \left(Y_t - Y_t^* \right) / Y_t^* + \varepsilon_t \tag{4}$$

where:

$$\beta_0 = \left(r^* - \alpha_1 \pi^* \right) \tag{5}$$

$$\beta_1 = \left(1 + \alpha_1 \right) \tag{6}$$

$$\beta_2 = \alpha_2 \tag{7}$$

As such, this exercise is a nice example of model specification and a lesson in identification.

The next step is to assemble the data on a new worksheet ply by copying the date, federal funds, inflation rate, and GDP gap data from the previous exercise. The Taylor rule parameter and correlation calculation rows are best deleted. Regression analysis is conducted in Excel by first installing the Analysis ToolPak Add in (Tools/Add ins/Analysis ToolPak) and then using the Tools/Data Analysis/Regression command. The dependent variable (federal funds rate) data is entered as the "Input Y Range", and the contiguous block of independent variable data as the "Input X Range." The results are best presented on a separate sheet. I like Excel to provide the residuals, and have students construct the appropriate plots themselves.

A nice aspect of this exercise is that students must manipulate the regression results to calculate the native Taylor rule parameter values from equations (5)-(7). For r^* and π^*, conditional values must be calculated for each (e.g. assuming the other has a value of 0.5). In addition, to test Taylor's hypothesized parameter values, the appropriate null value for the *t*-test is α_1 and α_2 = 0.5. Table 1 below displays a table results (generated in Excel):

[11] That being said, it should also be noted that the Fed was likely overestimating potential GDP at the time, and may have been rational in its bias.

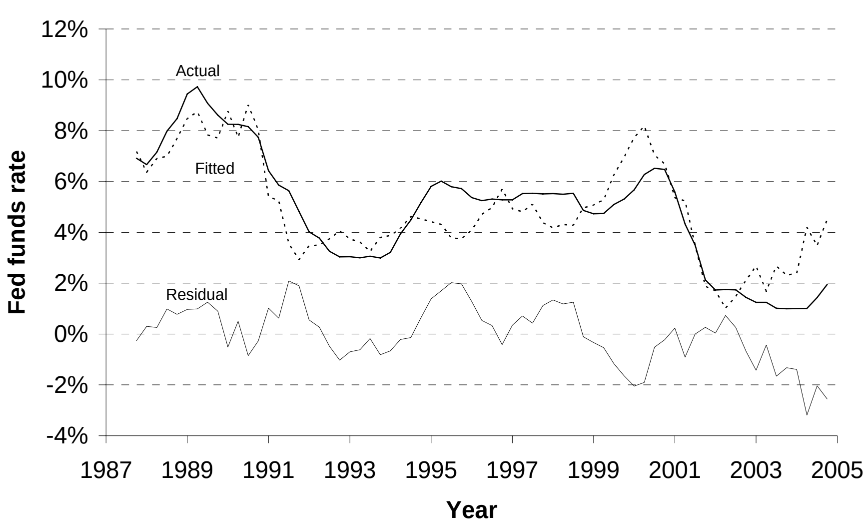
Table 1: Regression results summary

Parameter	Value	S.E.	t stat	p-value	t-stat	p-value
Full period			**Ho: =0**		**Ho: =.5**	
alpha 1	-0.203	0.067	-3.008	0.31%	10.424	0.00%
alpha 2	-0.031	0.094	-0.327	74.41%	-5.624	0.00%
r* (pi*=2%)	2.47%					
pi* (r*=2%)	4.31%					
R Square	0.507					
Burns						
alpha 1	-0.204	0.072	-2.850	0.78%	9.824	0.00%
alpha 2	0.539	0.081	6.671	0.00%	0.477	63.67%
r* (pi*=2%)	1.28%					
pi* (r*=2%)	-1.51%					
R Square	0.820					
Volcker						
alpha 1	-0.352	0.090	-3.890	0.05%	9.416	0.00%
alpha 2	-0.318	0.150	-2.112	4.34%	-5.437	0.00%
r* (pi*=2%)	5.11%					
pi* (r*=2%)	10.84%					
R Square	0.650					
Greenspan						
alpha 1	0.203	0.124	1.646	10.46%	2.398	1.93%
alpha 2	0.763	0.085	8.933	0.00%	3.082	0.30%
r* (pi*=2%)	2.03%					
pi* (r*=2%)	1.83%					
R Square	0.756					

Clearly, results are the most reasonable for the Greenspan years. A plot of actual values, fitted values, and residuals for this period is in Figure 6:

Figure 6:

Greenspan administration: 1987-present



In fact, the α_1 and α_2 values correspond fairly closely to the estimates generated by the Solver method earlier. The α_i values for nearly every other period are negative, which is a nonsensical result. For example, while a case can be made that the Burns administration accommodated inflation (giving a zero or negative α_1 value), the same cannot be said for Volcker. However, the conditional r^* values do correspond to the Judd and Rudebusch (1998) results: the Fed was over stimulating the economy during the Burns administration (consistent with a low r^* estimate) and generally contractionary during the Volcker administration (as suggested by the high r^* value). The reason the α_i estimates are so poor is likely that the Federal Open Market Committee practices interest rate target smoothing over time by adjusting values gradually to optimal (i.e. Taylor rule prescribed) targets. Judd and Rudebusch (1998) found evidence for smoothing, and a smoothing regression model produced good results.

Conclusion

The projects detailed in this paper provide a number of benefits to students. Primarily, they allow students to feel the power of estimating key macroeconomic relationships themselves. Perhaps, as a result, they will remember them better. However, along they way, they learn a number of lessons about data gathering, manipulating data into the form needed to estimate the relationships, manipulating theoretical relationships into estimable forms, and conducting hypothesis tests. They also learn some valuable tools in Excel that they can apply to other classes and in later careers.

Perhaps the best lesson that can be learned from these exercises is how to identify a reasonable estimate (e.g. those obtained in the Okun's Law example), and how to conduct appropriate hypothesis tests (e.g. identifying α_2 in the Taylor rule regression example). McCloskey and Ziliak (1996) reviewed a number of articles printed in *The American Economic Review* and noted that a large number of published studies had made fundamental econometric errors, including ignoring the real-world, or "economic significance" of the magnitude of coefficients, and reporting results of meaningless statistical tests. It is hoped that the examples in this paper provide opportunities outside an econometrics class to practice some aspects of good empirical analysis.

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Appendix A: Derivation of growth rate form of Okun's Law

The original form of Okun's Law is:

$$\omega(U^{*}-U) = (Y - Y^{*})/Y^{*} \tag{A1}$$

To derive the growth rate version, first expand equation (A1):

$$\omega U^{*} - \omega U = Y/Y^{*} - Y^{*}/Y^{*} \tag{A2}$$

Now totally differentiate equation (A2) with respect to all variables. However, treat the Y^{*} in the denominator on the right hand side as a constant. This is cheating a bit, but the result will be approximately right.

$$\omega dU^{*} - \omega dU = dY/Y^{*} - dY^{*}/Y^{*} \tag{A3}$$

If the natural rate of unemployment is unchanging, $dU^{*} = 0$. Again, this might not be strictly accurate, but we will use this to simplify our analysis. Using this assumption, and rearranging, gives us:

$$dY/Y^{*} = -\omega dU + dY^{*}/Y^{*} \tag{A4}$$

Usually, Y will be close to Y^{*} - within a few percent, at least. Therefore, we can approximate dY/Y^{*} with dY/Y, the growth rate of GDP. While this is another cheat, this should be accurate in the long run:

$$dY/Y = -\omega dU + dY^{*}/Y^{*} \tag{A5}$$

This concludes the derivation.

Appendix B: Okun's Law lab assignment

Lab assignment

For convenience, here is the growth rate version of Okun's Law again:

$$dY/Y = -\omega dU + dY^*/Y^* \qquad (2)$$

Since we have data for dU and dY/Y, we can estimate the other parameters. Equation (2) is simply the equation of a line ($y = mx + b$), with $y = dY/Y$, $x = dU$, slope $m = -\omega$, and intercept $b = dY^*/Y^*$. If we plot dU vs. dY/Y, we should be able to estimate the line that best goes through these points. Technically, this is called a "regression line." We are going to use Excel to estimate this line for us; Excel calls the line a "trendline."

Download the Excel spreadsheet for this exercise, and open it.

- The spreadsheet contains quarterly data for the change in unemployment rate and real GDP growth:
  - change in unemployment rate is measured as the unemployment rate in the last month in the quarter less the unemployment rate the previous year
  - real GDP growth is the percentage change since the previous year

1. Plot the dU and dY/Y data for 1949-1972 on a scatter plot chart. To accomplish this, highlight columns C and D by clicking (and hold down button) on the C button at the top of the column and moving your mouse pointer over the D button. Let go of the mouse button, click on the Insert menu, and then on Chart. Choose XY Scatter, and the option with no lines. Click through the menus, inserting titles, etc. as you wish. When finished, add the chart as new sheet, and give it the name 1949-1972. You should notice a definite downward trend in the data.

2. Now fit a regression line through the data points by adding a "trendline." Also make the equation of the trendline visible. To accomplish this, click on the Chart menu, then on Add Trendline. You want a linear trendline. Then click on the options tab, and select the display equation name on chart option. Click on OK.
(a) What is the Okun's Law coefficient? That is, if the unemployment rate rises by 1%, how much does real GDP growth change relative to potential?
(b) What was the growth rate of potential GDP in 1949-1972 according to this estimate?

3. Repeat the above exercise for the other two time periods: 1973-1996, and 1997-2004.
(a) Did the Okun's Law coefficient change?
(b) How much did the growth rate of potential GDP change between the three time periods? Does this match the figures you have read (in the "Engel's Law" reading) and we have discussed in class?

4. Aside from the assumptions and mathematical cheats, can you think of any weaknesses in the approach used in this exercise?

Appendix C: Taylor rule Replication lab

Read the Taylor (1993) paper. This lab will simulate the results of the policy rule in Taylor (1993), now known as the Taylor rule. Our goal is to simulate the equation written in this way:

$$i_t^T = r^* + \pi_t + \alpha_1\left(\pi_t - \pi^*\right) + \alpha_2\left(Y_t - Y_t^*\right)/Y_t^*$$

where i is the federal funds rate, T denotes target, π is the inflation rate, π^* is the inflation rate target, Y is GDP, and Y^* is potential GDP, and α_1 and α_2 are the policy parameters.

Initial questions

1. What are the parameter values for r^*, π^*, α_1 and α_2 chosen by Taylor?
2. What data series are needed? What time periods? (What options do we have?)
3. What is the frequency (time interval) of the data needed, or possible? (Are there choices?)
4. What units should be used for the variables?
5. What adjustments must be made to the data (to adjust for inflation, etc.)?

Goals for the lab

1. Collect necessary data from *FRED II* (Federal Reserve Economic Data version 2) from the Fed-St. Louis (http://research.stlouisfed.org/fred2/)
2. Format the data for the study
   - choose appropriate time period
   - put in appropriate units (decimal, percent, etc.; nominal or real with right base year)
   - choose appropriate frequency time interval
   - make appropriate calculations
   - collect on single sheet
   - also calculate summary statistics (mean, standard deviation) for all variables
3. Use Excel to calculate Taylor rule interest rate target for each time period
4. Chart results – TR rate vs. actual rate over time
5. Do correlation analysis
   - chart and analyze scatter plot of TR rate vs. actual rate, create "trendline" regression line
   - calculate correlation (r) (using =CORREL function)
   - tests hypotheses: H_0: r=0/ H_1: $r\neq0$, H_0: r=1/ H_1: r<0
   - look at correlation for Greenspan sub-time period
   - do a correlation test to compare to whole sample

Start writing a short paper

1. Write up summary of Taylor (1993) paper, focusing on the rule itself
2. Describe our analysis: equation, data used, etc.
   - type equation in Word
   - construct table to describe summary statistics (mean, std. dev.) in Word
3. Present results
   - incorporate nicely formatted chart into Word
   - present correlation results in text
4. Draw conclusions
   - can you think of any other tests or variations that could be performed?

Appendix D: Taylor rule Regression Lab

Introduction

This lab will use regression analysis to estimate the Taylor rule using the data you collected in Lab 1. This is partly based on Judd and Rudebusch (1998).

To remind you, the Taylor rule equation is:

$$i_t^T = r^* + \pi_t + \alpha_1\left(\pi_t - \pi^*\right) + \alpha_2\left(Y_t - Y_t^*\right)/Y_t^*$$

Where π usually refers to inflation over the previous year (four quarters).

Model 1

An obvious regression model is to regress the following:

$$i_t^T = \beta_0 + \beta_1\pi_t + \beta_2\left(\pi_t - \pi^*\right) + \beta_3\left(Y_t - Y_t^*\right)/Y_t^*$$

where we expect $\beta_0 = r^*$, $\beta_1 = 1$, $\beta_2 = \alpha_1$, and $\beta_3 = \alpha_2$. We would like to also estimate the value of π^*, but this would give us two constant terms in the regression. It would be impossible to tell them apart.

How would we run a regression on this model? On a worksheet, copy data in columns, so first column has *i* values, 2nd column has π values, 3rd column has $\pi - \pi^*$ values (you must make this column with a formula) and the 4th column has $(Y - Y^*)/Y^*$. Regress the equation for the period 1987 Q4-2004 Q3, the Greenspan period. Lab 1 suggested the fit is best in this period. To run the regression in Excel, use the Tools/Data Analysis…/Regression command. (You may have to install the Add-in Analysis Tool Pack) Select the data as required. Check the following options: "Confidence level 95", "New Worksheet ply" (give a name), and "Residuals". If you were to do this, what would happen?

The regression would not work. The problem is known as perfect multicollinarity. The π and $(\pi - \pi^*)$ variables are nearly identical, since π^* is a constant. As a result, the regression is invalid. If you think about it, how is one to determine the values of coefficients β_1 and β_2 if their variables are the same? It is simply impossible. As a result, we must rewrite the regression equation to avoid redundancies.

$$i_t^T = r^* + \pi_t + \alpha_1\pi_t - \alpha_1\pi^* + \alpha_2\left(Y_t - Y_t^*\right)/Y_t^*$$

$$i_t^T = r^* - \alpha_1\pi^* + \left(1+\alpha_1\right)\pi_t + \alpha_2\left(Y_t - Y_t^*\right)/Y_t^*$$

Now we can form the regression equation:

$$i_i = \beta_0 + \beta_1\pi_t + \beta_2\left(Y_t - Y_t^*\right)/Y_t^* + \varepsilon_t \tag{8}$$

where:

$$\beta_0 = \left(r^* - \alpha_1\pi^*\right) \tag{9}$$

$$\beta_1 = \left(1+\alpha_1\right) \tag{10}$$

$$\beta_2 = \alpha_2 \tag{11}$$

To run this regression, delete the $(\pi - \pi^*)$ column from the data, and run the regression again. Use a new worksheet ply name. Now it will work! But we can't individually identify r^* and π^*.

Goals

- Interpret reported t statistics. Are the reported t stats the appropriate tests to make? (No)
- Add a few rows under the regression results (before the residuals) to compute α_1, α_2, r^* given $\pi^* = 2\%$ and π^* given $r^* = 2\%$
- Do the appropriate tests: $t = \dfrac{\left(\hat{\beta} - \beta\right)}{s_{\hat{\beta}}}$ where s is the standard error, and a "hat" implies estimated value, and the plain β is hypothesized value. Excel reports the standard error for you.
  Use =TDIST(t, df, $tails$) to find the p –value. The degrees of freedom = n - # coefficients estimated (in this case the # of coefficients = 3).
- Construct a chart of predicted, residual, and actual values. From the residual data provided by Excel
  - Copy the year.quarter names down through the observations numbers using "Paste special/Values"
  - Form a column for actual values. You can do this by copying the data from the data ply, or by using the equation formula Actual = Predicted + Residual (since residual = actual – predicted)
  - Plot a scatter plot, and make it look nice.
- Is there anything usual about the residuals? (Yes, but do not write this up.)

Paper assignment

Your paper should have all of the sections of a regular paper, with appropriate headings. The total length should be about 5 pages.

- Write up the results from Lab 1: summarize Taylor (1993) as the lit review, write the equation in the equation editor, provide summary statistics of the data, include the comparison over time chart, write up hypothesis test for whether the correlation is statistically significantly different from zero. (H_0: r=0)
- Add summary Judd and Rudebusch (1998) to the literature review
- Add a summary of what we did in class to the literature review – pretend it is a published paper.
- Conduct and write up a regression analysis like the one completed in the lab for the following time period:
  - [student names]: Full period (1970 Q1 – 2004 Q4)
  - [student names]: Burns (1970 Q1 – 1978 Q1)
  - [student names]: Volcker (1979 Q3 – 1987 Q2)
- Calculate α_1, α_2 and π^* given r^*=2% for discussion in lab class on [date].
- Incorporate clean actual/estimated/residual chart of the assigned period into your paper.
- Conduct an analysis (hypothesis test) to determine whether the parameter values in your assigned time period correspond to those chosen by Taylor. Because this is only practice, only write up the following hypothesis test: in your time period, is the estimated α_1 statistically different from the value chosen by Taylor?
  - You should formally describe the test, and describe (and/or defend) all aspects of it: the hypothesized value, the statistic, degrees of freedom, number of tails, and p-value. Come see me if you need any help.
- Feel free to help each other on this paper, but construct the results (and write up the paper) yourselves.