Chain-weight GDP calculations
Student Instruction Sheet

Introduction
In this exercise, we will calculate chain-weight real GDP figures from raw data. Formally, chain-weight real GDP (Y_t) for year t is calculated from the prices (p) and quantities (q) of goods at certain time periods (t):

$$Y_t = \sqrt{\left(\frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}}\right)\left(\frac{\sum p_t q_t}{\sum p_t q_{t-1}}\right)} \times Y_{t-1}$$

Where the summation operator works over the implied index of goods. Note that the figure for year t depends on the figure for year ($t - 1$), and year ($t - 1$) GDP depends on year ($t - 2$), etc. in a "chain" back to the base year. Because of this chain, real GDP is most easily calculated using a multi-step process. Note that the first term under the square root is a Laspeyres relative quantity index (1+growth rate), and the second term is a Paasche relative quantity index. We first calculate these individual indexes (in growth rate form) for each year, and later calculate GDP levels.

1. For reference and later use, calculate nominal GDP for each year:
$$PY_t = \sum p_t q_t$$

2. For each year (except the first year) calculate the growth rate of real GDP using the current year's prices (this is a Paasche index):
$$\dot{Y}_t^P = \frac{\sum p_t q_t}{\sum p_t q_{t-1}} - 1$$

3. For each year (except the first year) calculate the growth rate of real GDP using the previous year's prices (this is a Laspeyres index):
$$\dot{Y}_t^L = \frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}} - 1$$

4. For each year (except the first year) calculate the chain-weight growth rate of real GDP by taking the geometric average of the two fixed-weight growth rates calculated in the previous two steps. The geometric average is the mathematically correct method of taking the average of growth rates.
$$\dot{Y}_t = \sqrt{\dot{Y}_t^P \cdot \dot{Y}_t^L}$$

5. Declare any year a base year (b). This means you are declaring that real GDP equals nominal GDP for that year:
$$Y_b = PY_b$$

6. Calculate the level of chain-weight real GDP for the year after the base year by using the base year and one plus the chain-weight growth rate computed in step 4:
$$Y_{b+1} = Y_b \cdot \left(1 + \dot{Y}_{b+1}\right)$$

7. Calculate the level of chain-weight real GDP for all successive years the same way: for year ($b + 2$), use Y_{b+1} and $\dot{Y}_{b+2}$, etc.

8. Calculate the level of chain-weight real GDP for the year before the base year rearranging the formula used in step 6, taken back one period:

$$Y_b = Y_{b-1} \cdot \left(1 + \dot{Y}_b\right) \Rightarrow Y_{b-1} = \frac{Y_b}{\left(1 + \dot{Y}_b\right)}$$

9. Calculate the level of chain-weight real GDP for each previous year the same way: for year (b - 2), use Y_{b-1} and $\dot{Y}_{b-1}$, etc.

Instructions
The goal of this assignment is to calculate chain-weight real GDP on an Excel spreadsheet for years 1-5 using year 3 as the base year given the following data:

		Year			
	1	2	3	4	5
Prices					
Good 1	$20	$21	$22	$23	$24
Good 2	$60	$59	$58	$57	$56
Quantities					
Good 1	75	76	77	78	79
Good 2	25	30	35	40	45

0. Enter the data into a spreadsheet. Include the labels as above, so "Prices" appears in cell A3, $20 appears in cell B4, etc. The following numbered steps correspond to the step numbers above.
1. Label the next row (row 9) "Nominal GDP", and calculate nominal GDP for each year using the formula above. Make sure to start the formula with an equal sign, and use an asterisk (*) for multiplication. For example, for Year 1, in cell B9 you should type: =(B4*B7+B5*B8). If you entered the formula correctly, the cell should read 3000.
1a. Label the next row (row 10) "Chain weight GDP calculations," and rows 11 through 14 "Real growth rate of output (Paasche index)", "Real growth rate of output (Laspeyres index)", "Real growth rate of output (chain-weight)", and "Real chain-weight GDP (Year 3 = base year)" respectively.
2. In row 11, enter a formula to calculate the real growth rate of output for year 2 (cell C11) using the current year's prices based on the formula above. Use the cell addresses containing the data rather than typing in the numbers directly. Remember to use parentheses to separate the numerator and denominator, and use "/" as the division operator. The correct syntax is =(C4*C7+C5*C8)/(C4*B7+C5*B8)-1. If you entered the formula correctly, you should get 10.36%. [To change the cell format to percentage form, click on the "%" button on the standard Excel toolbar. To increase the number of decimal places, click the appropriate button on the toolbar (it appears next to the comma button).] Copy cell C11 to cells D11 through F11 to make the calculation for the remaining years.
3. In row 12, enter a formula to calculate the real growth rate of output for year 2 (cell C12) using the previous year's prices based on the formula above. If you entered the formula correctly, you should get 10.67%. Copy the cell formula through year 5 as you did above. Note that the Laspeyres index always gives a higher growth rate than the Paasche index. To understand why, consider the data for year 1 and 2. The price of good 1 rose, and the price of good 2 fell. What is important is the relative price of good 1 to good 2 fell. As a result, consumers relatively substituted away from good 1 and towards good 2. This is reflected in the fact that the percentage increase in quantity of good 2 is larger than for good 1. (The quantity of good 1 rose despite the higher price because of general economic growth – what matters is the relative percentage increase.) The Paasche and Laspeyres growth rate figures are each a weighted average of the growth of the two goods, using the prices of the goods as weights. The difference is the Paasche index uses current prices, where the price of

good 2 has fallen and the price of good 1 has risen. That means that the low growth of good 1 has a relatively strong weight, and the fast growth of good 2 has a low weight, so the overall growth rate is lower. For the Laspeyres index, the previous year's prices are used as weights, so the slow growth good 1 has a lower weight, and the fast-growth good 2 has a higher weight – the result is a higher figure. But which figure is correct? A reasonable compromise is to average the two (geometrically) – this is the chain-weight method. It is possible to demonstrate that the chain-weight index is also very close to an ideal index (see assignment for Figure 3).

4. In row 13, enter a formula to calculate the real chain-weight growth rate of output for year 2 using the equation above and the Excel SQRT square-root function. The formula should refer to the appropriate cells in rows 11 and 12. If you typed the formula correctly, the cell will read 10.51%. Copy the formula through year 5.
5. In row 14, declare year 3 the base year by setting real GDP (in row cell D14) equal to nominal GDP using a simple formula setting the value equal to the value in cell D9. The cell should read 3274.
6. Calculate the level of chain-weight real GDP for year 4 by creating an Excel formula based on the appropriate equation above. Make sure to use the cell addresses for base year (year 3) real GDP and the chain-weight growth rate of real GDP in year 4. The correct figure is 4032.3.
7. Copy the year 4 formula to year 5.
8. Calculate the level of GDP for year 2 by creating an Excel formula based on the appropriate equation above. Remember that this uses year 3's chain-weight GDP level and growth rate. If you typed the formula correctly, the cell should read 3408.2.
9. Copy the year 2 formula to year 1.

Other assignments
➢ Calculate fixed-weight GDP level and growth using year 3 as the base year, and compare results to the chain-weight results. Carefully chosen figures will reveal a bias such that the level of fixed-weight GDP is above chain-weight GDP for every year except the base year. The growth rate of fixed-weight GDP is lower than chain-weight before the base year, and higher after the base year.
➢ Calculate the chain-weight and fixed-weight price indexes using the deflator formula:

$$P_t = \frac{PY_t}{Y_t}$$

and compare the results. The price indexes exhibit the opposite bias as the quantity indexes.