Notes on the Welfare Costs of Real and Money Transfers*

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The standard problem of the theory of international trade is the transfer problem—the effects of a continuing transfer from one country to another—on the welfare of the country making the transfer (the "transferor"). This problem has traditionally, following the work of Samuelson, been dealt with in terms of the question of whether or not there is a presumption that the terms of trade will turn against or in favour of the country making the transfer, thus imposing a "secondary burden," or mitigating the effects of the transfer by a "secondary benefit," by comparison with the effects of the transfer at unchanged terms of trade. Moreover, in this analysis the transfer is customarily assumed fixed in terms of one

or the other commodity, i.e., it is a "real" transfer. Rather than approach the transfer via the presumptive movement of the terms of trade, however, it seems more useful to approach the question of welfare effect directly. Moreover, it seems more natural to assume a transfer fixed in terms of money, rather than in terms of a commodity; this approach, incidentally, introduces an additional concept of secondary burden or benefit, associated with the question of whether the transfer, and the associated transfer of purchasing power and demand for money from transferor to transferee, increases the demand for money and reduces the general price level or vice versa. The purpose of these notes is to present some approximative formulae for the welfare effect of a transfer, on the simplifying assumption that money demand is proportional to money expenditure.

We begin with a transfer specified in terms of the transferor's export good, abstracting from money and assuming a barter economy. (The transfer could easily be assumed to be fixed in terms of the transferor's import good, by expressing it as a value magnitude in terms of the export good.) Country 1 is the transferor and country 2 the transferee, quantities being measured in terms of country 1's export good and p being the price of the other good in terms of it, assumed to be equal to unity in the absence of the transfer. The equilibrium

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overall balance of payments of country 1 is

$$B_1 = X_1 - pX_2 - T = 0$$

where X represents exports of the subscripted country and $X_1 = X_2 = X$ in the absence of the transfer.

Differentiating from a zero transfer,

$$\frac{\partial p}{\partial t} = \frac{1 - m_1 - m_2}{X(\eta_1 + \eta_2 - 1)}$$

where the m's are marginal propensities to import and the η's elasticities of demand for imports. Note that $\eta_1 + \eta_2 - 1$ is the stability condition, assumed satisfied, and that the terms of trade turn against or in favour of the transferor according as $m_1 + m_2 \lessgtr 1$, the usual criterion.

The equilibrium value of country 1's imports is approximately

$$X_2 - m_1 dT + \frac{\partial X_2}{\partial p}\frac{\partial p}{\partial T}dT$$

$$= X_2 - m_1 T - \eta_1 \frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}T$$

(substituting T for dT is the amount of the transfer).

The primary welfare loss from the transfer is T and the secondary loss (or benefit if $\partial p/\partial T < 0$) approximately

$$(X - m_1 T)\frac{\partial p}{\partial T}T - \frac{1}{2}T^2\frac{\partial p}{\partial T}\frac{\eta_1(1 - m_1 - m_2)}{\eta_1 + \eta_2 - 1}$$

$$= (X - m_1 T)\frac{T}{X}\frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}$$

$$- \frac{1}{2}\frac{T^2}{X}\frac{\eta_1(1 - m_1 - m_2)^2}{(\eta_1 + \eta_2 - 1)^2}$$

Letting $t = T/X$, the total welfare loss as a proportion of initial trade volume is

$$t\left[1 + \frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1} - \left\{\frac{1}{2}t\frac{(1 - m_1 - m_2)}{(\eta_1 + \eta_2 - 1)^2}\right.\right.$$

$$\left.\left. \times 2m_1(\eta_1 + \eta_2 - 1)\right\} + \eta_1(1 - m_1 - m_2)\right]$$

Note that the first two terms reduce to $\eta_1' + \eta_2'/\eta_1 + \eta_2 - 1$, where η' is an income-compensated elasticity, while the third term must be positive for $m_1 + m_2 > 1$ (favourable terms of trade movement) so that (to this approximation) there must be a welfare loss even if the terms of trade turn in favour of the transferor. Note also that, for a transfer fixed in terms of the import good, $\bar{T}$, we may merely substitute for t in the foregoing formula

$$t - \frac{\bar{t}}{1 - A\bar{t}},$$

where

$$A = \frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}.$$

We now turn to a monetary economy with the transfer fixed in terms of money, for simplicity assuming that each country is completely specialized, outputs being Y_1 and Y_2. Let π be the world price of country 1's good, and k_1 and k_2 the (assumed constant) money to expenditure ratios in the two countries, expenditure being output plus or minus the transfer. Let M be the fixed total of world money, equal to $k_1 Y_1 + k_2 Y_2$ so that $p = 1$ and $\pi = 1$ when $T = 0$, π being the money price of Y_1 and $T' = \pi T$ the money transfer. The demand for real balances (measured in country 1's good) is

$$\frac{M}{\pi} = k_1 Y_1 + k_2 Y_2 + (k_2 - k_1)T + k_2 Y_2 \frac{\partial p}{\partial T}T$$

$$\pi = 1 - T'\frac{k_2 - k_1 + k_2 \frac{Y_2}{X}\frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}}{k_1 Y_1 + k_2 Y_2}$$

$$T = \frac{T'}{1 - \frac{T'}{k_1 Y_1 + k_2 Y_2}\left(k_2 - k_1 + k_2 \frac{Y_2}{X}\frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}\right)}$$

$$\frac{T}{X} = \frac{t'}{1 - t'\frac{X}{k_1 Y_1 + k_2 Y_2}\left(k_2 - k_1 + k_2 \frac{Y_2}{X}\frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}\right)}$$

where $t' = T'/X$ is the ratio of the money transfer to the value of exports at initial pre-transfer money prices. (Note that differences between the marginal and the initial average money-to-expenditure ratios could be allowed for by priming k_1 and k_2 in the parenthetical expression in the denominator).

Letting $\frac{1 - m_1 - m_2}{\eta_1 + \eta_2 - 1}$ be represented by A, and $\frac{X}{k_1 Y_1 + k_2 Y_2}\left(k_2 - k_1 + k_2 \frac{Y_2}{X}A\right)$ by B, the welfare loss as a proportion of the initial value of trade is

$$\frac{t'}{(1 - t'B)^2}$$

$$\left[1 + A - t'\left(B + AB + m_1 A + \frac{1}{2}\eta_1 A^2\right)\right]$$

or

$$t'\left[1 + A - \frac{t'}{(1 - t'B)^2}\left(t'B^2 + t'AB^2\right.\right.$$

$$\left.\left. - B - AB + m_1 A + \frac{1}{2}\eta_1 A^2\right)\right]$$

as compared with

$$t\left(1 + A - m_1 tA - \frac{1}{2}t\eta_1 A^2\right)$$

for the transfer specified in export goods. The welfare loss as a proportion of the transferor's initial national income and expenditure can easily be obtained by multiplying by X/Y_1. The compound parameter B can be rewritten

as

$$B = \frac{(k - 1)x + kyA}{1 + ky},$$

where $k = k_2/k_1$, $x = \frac{X}{Y_1}$ and $y = \frac{Y_2}{Y_1}$, and these symbols represent respectively the velocity of circulation of money in country 1 relative to country 2 (the money-to-expenditure ratio in country 2 relative to country 1), the importance of trade to country 1, and the size of the rest of the world relative to country 1.

The main purpose of approaching the effects of transfers in terms of (approximative) welfare cost rather than presumptive movement of the terms of trade is to gain some quantitative idea of the influence of the various parameters suggested as relevant by economic analysis but ignored by calculations based on the assumed constancy of prices in face of the transfer. Table I is a preliminary calculation which ignores the presence of money and compares the welfare costs of transfers fixed in terms of the export and the import good when the conditions for the "classical presumption" that the terms of trade turn against the transfer are fulfilled. It should be noted that in the case of a "real" transfer the sizes of the two countries relative to each other and to the initial volume of trade between them is irrelevant, unless it systematically influences the parameters themselves, since trade volume is the same for both countries. The attempt to derive presumptions from consideration of relative sizes was, as Samuelson's classical articles on the transfer problem showed, a blind alley, into

TABLE I

WELFARE COST OF TRANSFER FIXED IN REAL TERMS, AS PERCENTAGE OF
PRE-TRANSFER TRADE VOLUME (t FOR EXPORT GOOD, $\bar{t}$ FOR
IMPORT GOOD, m AND η SAME FOR BOTH COUNTRIES)

m	η	$t = 10\%$	$\bar{t} = 10\%$	$t = 25\%$	$\bar{t} = 25\%$	$t = 50\%$	$\bar{t} = 50\%$
.45	0.75	11.895	12.136	29.344	30.652	57.375	63.426
	1.00	10.950	11.060	27.187	27.876	53.750	56.510
	1.25	10.634	10.705	26.462	26.904	52.514	54.314
	1.50	10.476	10.528	26.098	26.412	51.891	53.205
	2.00	10.317	10.351	25.733	25.948	51.264	52.125
.40	0.75	13.780	14.345	33.625	37.192	64.500	78.906
	1.00	11.900	12.141	29.375	30.887	57.500	63.580
	1.25	11.269	11.420	27.931	28.878	55.056	58.864
	1.50	10.953	11.063	27.203	27.893	53.813	56.579
	2.00	10.636	10.707	26.472	26.918	52.556	54.340
.35	0.75	15.655	16.630	37.844	44.075	71.375	96.684
	1.00	12.850	13.242	31.563	34.039	61.250	71.281
	1.25	11.905	12.146	29.406	30.921	57.625	63.735
	1.50	11.431	11.603	28.316	29.402	55.766	60.135
	2.00	10.955	11.065	27.219	27.909	53.875	56.649
.30	0.75	17.520	18.999	42.000	51.563	78.000	116.667
	1.00	13.800	14.367	33.750	37.346	65.000	79.688
	1.25	12.542	12.795	30.889	33.036	60.222	68.935
	1.50	11.910	12.151	29.438	30.956	57.750	63.889
	2.00	11.276	11.427	27.972	28.924	55.222	59.056
.25	0.75	19.375	21.451	46.096	59.722	84.375	137.500
	1.00	14.750	15.512	35.938	38.091	68.750	88.889
	1.25	13.181	13.630	32.378	35.228	62.835	74.500
	1.50	12.391	12.705	30.566	32.555	59.766	67.857
	2.00	11.597	11.792	28.733	29.962	56.597	61.570

which the English neo-classicals were led in part by their attempt to relate demand to utility, assumed to be cardinal but more fundamentally by their mistaken assumption that the transfer must be effected by shifts along given demand curves, and the terms of trade must turn against the transferor. As will appear from the formulas presented above, relative sizes do become relevant for the case of a transfer fixed in money terms, because relative sizes determine the size of the shift in the weights attached to national demands for money resulting from the transfer, and therefore the movement of the world price level.

The figures in the table can be used to study four questions that have appeared in the literature concerning the influence on the size of the "secondary burden" of the transfer of variations in the elasticities of international demand, differences in marginal preferences for goods, the size of the transfer (as a proportion of initial trade) and the choice between denomination of the transfer in exportable and in importable goods. In connection with the first two variables, it should be noted that, since the elasticity of demand for imports can be decomposed into a marginal propensity to import and a compensated elasticity of demand, use of the uncompensated elasticity and the marginal propensity to import as parameters implies an equal and opposite change in the compensated elasticity when the marginal propensity to import changes (a reduction in the propensity representing an increase in the difference of marginal tastes). The use of the uncompensated rather than the compensated elasticity is chosen for familiarity of concepts for purposes of discussion, and the alternative approach can easily be developed by the interested reader; its main effect is to understate the effect of increasing difference in marginal preferences in increasing the "secondary burden."

The main results shown by the table are two. First, the "secondary burden" becomes significant relatively to the "primary burden" only when taste differences are large *and* demand elasticities are low, and significant in absolute terms only when the transfer is large as well. Note in particular the figures associated with the maximum import demand elasticity of two shown in the table: recent empirical studies tend to suggest that a value of two is rather low. Second, and rather more surprising in view of the importance that theory has attached to the difference between transfers fixed in terms of exportable and of importable goods, the difference between the welfare costs of the two specifications are negligible unless the transfer is very large (in relation to initial trade), the elasticities of import demand very low, and the marginal taste differences very large. Consider the figures for a transfer of 25 per cent of initial trade, elasticities of 1.5, and marginal propensities to import of .35. The results would indeed suggest that concentration on the question of whether the transfer is denominated in exportable or in importable goods is justified only in near-pathological cases.

Table II presents calculation of the (approximate) welfare costs of transfers fixed in money and expressed as a proportion of the initial trade volume (t') for values of .10, .25, and .50, on the assumptions that the rest of the world is alternatively one, two, and four times the size of the transferor (y being the ratio of foreign to domestic income); and the money-to-income ratio there is respectively one, one-and-a-half, and two times the transferor's ratio (k being the ratio of the foreign to the domestic money-to-income ratio), for the values of the marginal propensities to import (m) and elasticities of demand for imports (η) previously imposed. It is assumed that before the transfer exports (and imports) are one quarter of the transferor's national income. This last assumption seemed the most reasonable one to make about the parameter; it was most convenient to fix in order to reduce the possibilities of parametric variation to a single, easy-to-read table.

The chief interest of Table II, as compared with Table I, concerns the effects of variations in monetary conditions on the (approximate) size of the secondary and therefore the total welfare cost of the transfer. This involves two dimensions, differences in the money-to-income ratios and differences in relative sizes of countries. As the table shows, the approximate welfare burden increases as the money-to-income ratio in the transferee rises relative to that of the transferor, and the size of the transferee increases relative to that of the transferor. (The numbers shown in the last line of Table II(a), for $k = 2$, however, may indicate that with small enough transfers, high enough elasticities and a large enough difference in marginal propensities *and* in money-to-income ratios, an increase in the relative size of the transferee reduces the welfare cost of the transfer; this possibility is not explored here.) However, the increases in welfare burden as the money-to-income ratios and relative national income differences increase are very small relative to the size of the burden (the welfare cost) for equal incomes and money-to-income ratios, set by the magnitudes of the elasticities and the marginal taste difference. This in turn suggests that analysis may fairly safely concentrate on the real parameters to the neglect of the monetary ones, subject to the necessity of recognizing that in cases requiring large changes in relative prices atten-

TABLE II

APPROXIMATE WELFARE COSTS OF A MONEY TRANSFER EXPRESSED AS A PROPORTION OF INITIAL TRADE WITH TRADE INITIALLY ONE QUARTER OF TRANSFEROR'S INCOME, FOR VARIOUS VALUES OF THE MARGINAL PROPENSITY TO IMPORT, ELASTICITY OF DEMAND FOR IMPORTS, RELATIVE SIZE OF REST OF WORLD, AND RATIO OF FOREIGN TO DOMESTIC MONEY-TO-INCOME RATIO

m	η	y = 1			y = 2			y = 4		
		k = 1.0	k = 1.5	k = 2.0	k = 1.0	k = 1.5	k = 2.0	k = 1.0	k = 1.5	k = 2.0
(a) $t' = .10$										
.45	0.75	11.989	12.099	12.156	12.054	12.113	12.148	12.087	12.122	12.142
	1.00	11.005	11.071	11.116	11.023	11.067	11.094	11.038	11.064	11.079
	1.25	10.670	10.730	10.771	10.681	10.721	10.745	10.691	10.714	10.727
	1.50	10.502	10.560	10.599	10.511	10.553	10.571	10.618	10.540	10.522
	2.00	10.334	10.390	10.427	10.340	10.375	10.390	10.345	10.365	10.376
.40	0.75	14.057	14.185	14.272	14.151	14.244	14.301	14.228	14.287	14.320
	1.00	12.013	12.104	12.161	12.060	12.118	12.154	12.092	12.128	12.148
	1.25	11.344	11.417	11.469	11.369	11.418	11.447	11.401	11.420	11.436
	1.50	11.008	11.074	11.119	11.026	11.070	11.087	11.041	11.067	11.082
	2.00	10.672	10.732	10.773	10.683	10.723	10.747	10.693	10.716	10.918
.35	0.75	16.128	16.309	16.432	16.292	16.428	16.511	16.426	16.514	16.564
	1.00	12.991	13.149	13.220	13.109	13.184	13.229	13.162	13.209	13.235
	1.25	12.024	12.109	12.167	12.065	12.123	12.158	12.097	12.133	12.153
	1.50	11.517	11.592	11.643	11.546	11.597	11.627	11.569	11.600	11.617
	2.00	11.010	11.076	11.121	11.028	11.072	11.099	11.043	11.069	11.084
.30	0.75	18.229	18.472	18.638	18.479	18.666	18.780	18.683	18.807	18.876
	1.00	14.077	14.205	14.293	14.172	14.266	14.322	14.249	14.307	14.342
	1.25	12.710	12.808	12.876	12.767	12.836	12.839	12.813	12.856	12.880
	1.50	12.029	12.114	12.172	12.070	12.128	12.163	12.102	12.138	12.158
	2.00	11.351	11.425	11.473	11.377	11.425	11.455	11.397	11.426	11.443
.25	0.75	20.360	20.675	20.891	20.711	20.959	21.111	21.007	21.167	21.261
	1.00	15.122	15.276	15.380	15.250	15.364	15.433	15.354	15.427	15.468
	1.25	13.402	13.515	13.592	13.477	13.556	13.607	13.538	13.589	13.617
	1.50	12.546	12.642	12.706	12.599	12.666	12.706	12.642	12.683	12.706
	2.00	11.694	11.773	11.826	11.727	11.780	11.812	11.753	11.785	11.809
(b) $t' = .25$										
.45	0.75	30.079	30.616	30.985	30.332	30.704	30.932	30.538	30.767	30.896
	1.00	27.527	27.950	28.234	27.642	27.920	28.089	27.735	27.401	27.994
	1.25	26.683	27.066	27.329	26.757	27.006	27.158	26.817	26.964	27.046
	1.50	26.261	26.627	26.877	26.316	26.552	26.695	26.360	26.500	26.575
	2.00	25.840	26.190	26.428	25.877	26.112	26.196	25.732	26.034	26.159
.40	0.75	35.277	36.137	36.705	35.922	36.526	36.898	39.627	36.808	37.028
	1.00	30.112	30.650	31.019	30.366	30.738	30.966	30.802	30.802	30.931
	1.25	28.398	28.855	29.194	28.557	28.864	29.052	28.685	28.871	28.975
	1.50	27.544	27.964	28.251	27.659	27.969	28.106	27.752	27.918	28.011
	2.00	26.693	27.077	27.339	26.767	27.017	27.169	26.827	26.974	27.056
.35	0.75	40.768	41.939	42.758	41.830	42.734	43.295	42.720	43.320	43.661
	1.00	32.647	33.428	33.892	33.172	33.655	33.951	33.514	33.374	33.991
	1.25	30.144	30.684	31.054	30.399	30.772	31.001	30.605	30.751	30.965
	1.50	28.849	29.327	29.654	29.031	29.355	29.553	29.178	29.375	29.485
	2.00	27.560	27.981	28.268	27.760	27.954	28.124	27.769	27.935	28.028
.30	0.75	46.296	47.887	49.009	47.929	49.203	49.401	49.320	50.185	50.682
	1.00	35.457	36.283	36.855	36.065	36.674	37.050	36.567	36.959	37.181
	1.25	31.927	32.555	32.998	32.288	32.734	32.751	32.583	32.863	33.020
	1.50	30.178	30.719	31.090	30.433	30.807	31.036	30.640	30.786	31.001
	2.00	28.440	28.899	29.214	28.599	28.908	29.097	28.728	28.915	29.020
.25	0.75	51.916	54.133	55.622	54.376	56.104	57.194	56.397	57.600	58.291
	1.00	38.223	39.220	39.914	39.050	39.804	40.270	39.738	40.232	40.512
	1.25	33.742	35.606	34.970	34.211	34.749	35.072	34.618	34.952	35.141
	1.50	31.529	32.137	32.555	31.864	32.292	32.555	32.137	32.404	32.555
	2.00	29.335	29.835	30.178	29.542	29.883	30.667	29.708	29.917	30.034
(c) $t' = .50$										
.45	0.75	60.249	62.438	63.988	61.686	62.805	63.762	62.116	63.070	63.612
	1.00	55.095	56.801	57.999	55.559	56.692	57.394	55.935	56.613	56.997
	1.25	53.390	54.948	56.038	53.689	54.702	55.328	53.930	54.527	54.864
	1.50	52.540	54.027	55.066	52.760	53.716	54.306	52.937	53.496	53.773
	2.00	51.691	53.108	54.052	51.836	52.736	53.126	51.951	52.473	52.767
.40	0.75	70.988	75.032	76.768	73.448	75.995	77.609	75.539	77.216	78.181
	1.00	60.388	62.588	64.145	61.416	62.956	63.918	62.264	63.223	63.708
	1.25	56.901	58.765	60.160	57.538	58.802	59.587	58.062	58.829	59.264
	1.50	55.162	56.872	58.388	55.626	56.762	57.466	56.004	56.684	57.068
	2.00	53.934	54.994	56.086	53.733	54.747	55.375	53.974	54.572	54.910
.35	0.75	82.180	86.982	90.496	86.523	90.393	92.878	90.331	92.990	94.534
	1.00	68.888	68.698	70.638	67.593	69.619	70.894	69.021	70.254	71.065
	1.25	60.526	62.737	64.303	61.559	63.107	64.074	62.411	63.374	63.923
	1.50	57.869	59.823	61.323	58.606	59.941	60.766	59.208	60.024	60.487
	2.00	55.227	56.941	58.145	55.693	56.831	57.537	56.071	56.793	57.138
.30	0.75	93.750	100.236	105.037	100.413	105.886	108.434	106.401	110.274	112.550
	1.00	71.605	75.032	77.502	72.781	75.277	78.362	76.247	77.514	78.946
	1.25	64.298	66.879	68.769	65.764	67.636	67.711	66.997	68.191	68.530
	1.50	60.665	62.886	64.460	61.703	63.259	64.230	62.559	63.527	64.078
	2.00	57.075	58.954	60.277	57.720	58.992	59.782	58.247	59.019	59.457
.25	0.75	105.556	113.855	120.000	114.844	122.025	126.654	123.264	128.395	131.361
	1.00	77.551	81.726	84.765	81.000	84.251	86.368	83.984	86.194	87.471
	1.25	68.168	71.186	73.351	69.518	72.387	73.799	71.822	73.270	74.101
	1.50	63.556	66.069	67.858	64.928	66.729	67.858	66.069	67.208	67.858
	2.00	58.979	61.034	62.485	59.818	61.234	62.116	60.507	61.377	61.872

tion must be paid to the division of the relative price change between a fall in the money price of exportables and a rise in the money price of importables, and this division will depend on relative money-to-income ratios and national income sizes. For given elasticities and marginal propensities to import, and equal money-to-income ratios, the welfare cost of a transfer fixed in terms of money must (and in the figures of Table II does) fall between the welfare costs of transfers fixed in terms of exportables and importables, and can be approximated by either as a limit, though with low elasticities and large differences in marginal preferences an average of the two limits would be a significantly closer approximation.

This last consideration suggests a somewhat different type of problem, whose solution must await another occasion or possibly another author. Suppose that, contrary to the assumptions of the classical transfer analysis but in line with the spirit of Samuelson's classic investigations, we know only the "structure" of the countries involved in the transfer, that is, their sizes, average propensities to spend on imports and domestic goods, and money-to-income ratios, the marginal propensities being unknown; and suppose that the transferring country has a choice among the alternatives of denominating the transfer in terms of exportable goods, importable goods, and money. What numéraire should it choose, and how important would the choice be for its likely level of welfare after the transfer?