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The Contemporary Relevance of Marx's Economics: Theory and Evidence

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THEORETICAL FRAMEWORK

The purpose of this paper is to test Marx's predictions pertaining to the evolution of the capitalist production system in the long-run period in which technological changes can occur. Since these predictions of particular dynamic tendencies are falsifiable hypotheses, it is appropriate to test them for both logical consistency and empirical validity. This requires the aid of an adequate analytical model of capitalist production. The essential properties of such model were outlined partly by Marx himself, and partly in the contributions of subsequent economists, in particular Leontief and Sraffa.

In *Capital* Marx generalized the Ricardian *classical* analysis of production and distribution in a competitive capitalist economy.[1] The price formation and distributional aspects of classical economics were rigorously formalized for the first time at the turn of the century by Dmitriev in his *Economic Essays*. Dmitriev expressed the inverse relationship between the real wage and the rate of profit in the form of a mathematical function. His wage-profit function was rediscovered by Samuelson (1957), and his analysis of the price system was elaborated by Sraffa (1960). Leontief (1936, 1951) developed input-output analysis which was anticipated in *Capital* and Dmitriev's *Economic Essays*. A production model compatible with this Leontief-Marx-Sraffa tradition can be specified as follows.

A capitalist production system consists of a set of basic industries which produce material commodities by means of labour and material commodities that serve directly or indirectly as material inputs in the production of all other commodities. Production technology can be represented by the coefficient matrices B and R, and the coefficient vector L. $B \geq 0$ denotes an $n \times n$ matrix of fixed capital input coefficients per unit of gross output, $R \geq 0$ refers to an $n \times n$ diagonal matrix of replacement rates, and $L \geq 0$ is a row-n vector of labour input coefficients per unit of gross output. Define A, an $n \times n$ Leontief technology matrix, as $A = RB$.[2] The reduced form of representation of production technology is provided by the $(n + 1) \times n$ augmented matrix G consisting of A and L.

A technology is said to be feasible if the maximum eigenvalue of A is real, positive and less than unity. The set of all feasible, available technologies is denoted by T, $T = \{G_1, G_2 \ldots\}$. Given the uninterrupted growth of technical knowledge in a capitalist system, set T expands over time. (Marx, 1977, p.617)

Denoting the row-n vector of production prices by P, the wage rate by w and the rate of

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profit by r, the pricing regime can be expressed by the following system of linear equations:

(1) $$P = PA + rPA + wL.$$

Rearranging the terms in (1), and denoting the $n \times n$ unit matrix by I, we have

(2) $$P = rPA[I - A]^{-1} + wL[I - A]^{-1}.$$

The matrix $A[I - A]^{-1}$ represents integrated material requirements of production, whereas the vector of Marx's labour values $L[I - A]^{-1}$ expresses integrated labour requirements per unit of net output.

Define an $n \times n$ matrix M as $M = A[I - A]^{-1}$, and an $1 \times n$ vector V as $V = L[I - A]^{-1}$. The solution for long-run equilibrium prices can be expressed as

(3) $$P = wV[I - rM]^{-1}.$$

Choose some ${}_j$th commodity as a numeraire and assume that the real wage is determined exogenously by class conflict. The solution of the linear equations expressed by (3) then determines the relative price structure and the rate of profit.[3]

The equation system (3) contains an implicit polynomial function of n – th degree between the real wage and the rate of profit which can be expressed explicitly as

(4) $$w(r) = 1/V[I - rM]^{-1}E_j,$$

where E_j denotes the ${}_j$th $n \times 1$ unit vector. Each feasible technology determines a unique *wage-profit function.* Note that the graph of this function in the $r - w$ space is a strictly decreasing curve which shows income distribution possibilities associated with the underlying technology.

The intercepts of this curve are, respectively, the maximum real wage and the maximum rate of profit. Observe that the maximum real wage, w(0), is identical with the reciprocal of the ${}_j$th commodity's labour value, i.e. $w(0) = 1/v_j$. Note that the maximum rate of profit, r^*, is equal to the inverse of the maximum eigenvalue of M matrix, denoted by m, i.e. $r^* = 1/m$. This maximum eigenvalue is an increasing function of the elements of M, according to a Frobenius-Perron theorem concerning semipositive matrices.[4] The above analysis implies that the maximum real wage provides an index of the labour intensity of production, whereas the maximum rate of profit measures the material intensity of production.

Technological choice in a competitive capitalist economy is based on the *profit-maximization* criterion. The development of labour productivity and material intensity of the production process manifests itself in an historical shift pattern of the wage-profit curves corresponding to successive technologies in use. Production theory at this level of abstraction predicts no particular tendency of technological change in a capitalist economy. However, Marx argued that certain evolutionary tendencies are specific to the capitalist development of production technology.

First, he hypothesized in *Grundrisse* of 1857–58 that the capitalist mode of production would "reduce labour time for the whole society to a diminishing minimum. . . ." (Marx, 1973, p.708) This tendency towards full automation implies *labour-saving* technological change which would shift the vertical intercept of the wage-profit curve away from the origin, since $v_j(t) < v_j(0)\ \forall\ t > 0$, where t indexes time and 0 denotes the base year of observation.

Second, Marx claimed in diverse parts of *Capital* and elsewhere that large-scale industry tends to undermine the ecological base of the production process. (Romoren 1973, 175–86) This hypothesis implies a growing material intensity of production with the subsequent deterioration of the natural environment due to the increased waste flows per unit of output. Such a tendency would manifest itself in quantitative increases of elements in the Leontief technology matrix. Therefore, the maximum eigenvalue m would have to rise over time. Given the inverse relationship between the maximum rate of profit and m, a material-using tendency of technological change must result in an inward shift of the horizontal intercept of the wage-profit curve, because Marx's second hypothesis predicts that $m(t) > m(0)\ \forall\ t > 0$.

Third, Marx asserted in Part III of the last volume of *Capital* that the rate of profit would decline in the very long-run. As well known, he claimed that if technological change increases the organic composition of capital, "then this gradual growth in the constant capital, in relation to the variable, must necessarily result in a *gradual fall in the general rate of profit,* given that the rate of surplus-value, or the level of exploitation of labour by capital, remains the same." (Marx, 1981, p.318) To check the logical validity of Marx's assertion, we can utilize Sraffa's approach to the analysis of income distribution.

Sraffa (1960) specified a standard system which simplifies the analysis of income distribution by means of a linear wage-profit relation

(5) $$r = r^*(1 - w_s),$$

where w_s denotes the share of wages in the standard net product, and r and r^* refer respectively to the actual and maximum rates of profit. The relation (5) is also valid for the actual production system, provided only that the wage rate is expressed in terms of standard commodity. In the context of Sraffa's standard system, the rate of exploitation e can be defined as

(6) $$e = (1 - w_s)/w_s.$$

Since it follows from (5) that $w_s = 1 - (r/r^*)$, the rate of surplus value can also be expressed in terms of r and r^* as

(7) $$e = r/(r^* - r).$$

It follows that the rate of profit is given by

(8) $$r = r^*/(1 + 1/e).$$

The growing organic composition of capital in Marx's scenario must be reflected in a reduction of r^*. Assuming that e is constant, (8) clearly implies that the rate of profit must decline in this case.[5] However, there is no reason to expect *a priori* either that r^* will fall or that e will be constant over time. Even if r^* is reduced by technological change, an increase in e can neutralize the law of the falling rate of profit.[6] Marx was aware of this problem and consequently "viewed the falling rate of profit not as a mechanical necessity, but as a tendency which at times could be offset by counteracting tendencies." (Gillman, 1957, p.7) Thus the law of the falling rate of profit constitutes an *empirical* hypothesis which may be significant in specific periods of capitalist development.[7]

The assessment of the general theoretical relevance of the law of the falling rate of profit in the analysis of technological change continues to be one of the more controversial areas in Marxian economics. "Most Marxians, and others as well, hold that the predominant bias in capitalist development is toward 'capital-using, labour-saving' innovations, and all variations on the Marxian falling profit-rate theory are based on this supposition of a 'rising technical composition of capital.' " (Schutz, 1985, Pt II, p.8) Since a detailed discussion of the various arguments pertaining to this theory is beyond the scope of this paper, only a few observations relevant to the matter at hand will be made.

As stated earlier, production theory predicts no particular pattern of technological change in a profit-maximizing economy. Therefore, the widely used assumption of the 'capital-using, labour-saving' type of technological progress is questionable in a general production model. Further, even if this problematic assumption were warranted on empirical grounds, a logical deduction of the falling rate of profit theorem would require a well-behaved neoclassical economy with an inverse relationship between factor prices and factor intensities. However, the so-called Cambridge controversies in production and capital theory led to the undisputed conclusion that there is no reason to expect a systematic relationship of this type except in a purely hypothetical case of an economy which produces a single commodity. (Hodgson, 1974, pp.69–70) The law of the falling rate of profit cannot be derived from the logical structure of the contemporary production theory, although the rate of profit may actually decline in the long period.

The scenario of capitalist development resulting from Marx's three hypothetical propositions discussed above cannot be rejected on logical grounds alone. All three tendencies *may* prevail coincidentally, if one accepts the logic of the contemporary Leontief-Marx-Sraffa production model. Carried out in the framework of this model, the present analysis suggests that Marx's results may be empirically correct in *specific* cases, but cannot claim *general* theoretical validity. Only an empirical test can decide, whether the time path of capitalist development asserted by Marx actually did materialize in the course of economic history.

To conduct such a test, it is sufficient to estimate the actual wage-profit functions from a time series of observations of the available input-output data. Historical estimates of these functions must be obtained from input-output tables in constant prices. Otherwise, the pattern of shifts in the estimated wage-profit curves is unreliable, because price changes are bound to cause estimation bias. The confrontation of Marx's hypotheses with empirical results derived from Canadian data is presented in the following section.

EMPIRICAL FINDINGS

Numerical estimates of the Canadian wage-profit functions for five years of observation (1961, 1966, 1971, 1976 and 1980) are presented in Table 1 and shown graphically in Figure 1. The empirical validity of Marx's scenario of capitalist development can be ascertained from an inspection of Table 1 and Figure 1.

The wage-profit functions were estimated from the Canadian input-output data in constant prices which are described in the appendix. Since the available data are expressed in price terms instead of physical units of measurement assumed in the theoretical model, the estimated technical coefficients of year t are captured by the matrix $\hat{P}A(t)\hat{P}^{-1}$ and vector $L(t)\hat{P}^{-1}$, where $\hat{P}$ refers to a diagonal matrix of constant prices (Özol 1984, p.360) Therefore, the estimated wage-profit functions are equal to the true wage-profit functions multiplied by the constant price of the numeraire commodity. Consequently, the vertical intercepts of the estimated functions differ from the vertical intercepts of the true functions due to the presence of this multiplicative price factor. Note that the horizontal intercepts of the estimated functions are not affected by the price terms because $\hat{P}A(t)\hat{P}^{-1}$ is a similarity transformation of the true Leontief technology matrix A at time t and thus must have the same maximum eigenvalue as A(t). The shift pattern of the estimated wage-profit functions is obviously analogous to the shift pattern of the true functions.

TABLE 1
Numerical Estimates of the Canadian Wage-Profit Functions

r(%)	w(1961)	w(1966)	w(1971)	w(1976)	w(1980)
0.00	4.4160	7.5722	8.1549	9.1980	9.2394
10.00	4.3056	7.3592	7.9108	8.8714	8.8704
20.00	4.1868	7.1298	7.6483	8.5213	8.4756
30.00	4.0589	6.8829	7.3659	8.1461	8.0356
40.00	3.9209	6.6169	7.0619	7.7439	7.6027
50.00	3.7722	6.3300	6.7344	7.3125	7.1211
60.00	3.6115	6.0201	6.3809	6.8496	6.6069
70.00	3.4375	5.6847	5.9985	6.3525	6.0581
80.00	3.2489	5.3206	5.5836	5.8178	5.4725
90.00	3.0435	4.9237	5.1312	5.2415	4.8477
100.00	2.8190	4.4886	4.6347	4.6181	4.1811
110.00	2.5718	4.0077	4.0840	3.9399	3.4697
120.00	2.2973	3.4693	3.4632	3.1948	2.7097
130.00	1.9883	2.8545	2.7444	2.3613	1.8963
140.00	1.6330	2.1291	1.8733	1.3957	1.0221
150.00	1.2102	1.2230	0.7249	0.1917	0.0744
150.75	1.1751	1.1442	0.6209	0.0857	0.0000
151.34	1.1464	1.0810	0.5365	0.0000	
154.74	0.9751	0.6880	0.0000		
159.74	0.6911	0.0000			
160.00	0.6753				
169.07	0.0000				

Notes: Numeraire: agricultural output
The wage rate (w) is measured in thousands of 1971 Canadian dollars.

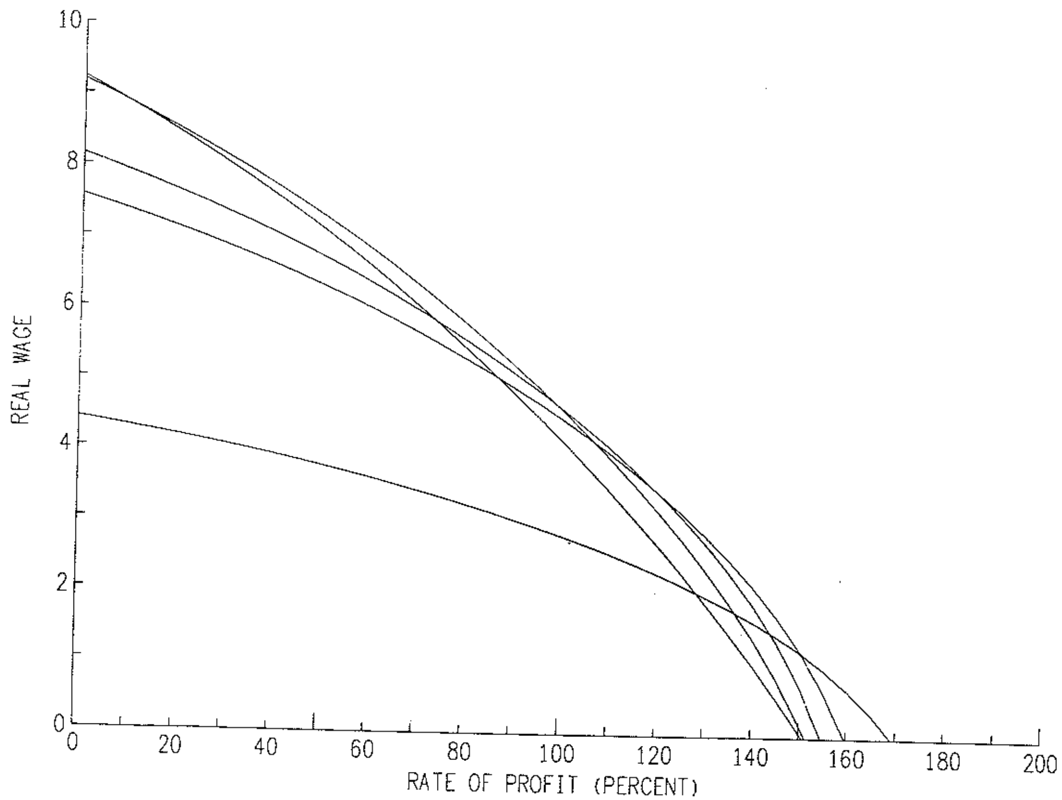

Figure 1. Historical Shifts of the Canadian Wage-Profit Functions

To illustrate, consider the vertical and horizontal intercepts of the wage-profit curve for 1961 in Table 1 and Figure 1. The vertical intercept is equal to $p_j/v_j(61)$, where p_j and $v_j(61)$ refer to the constant price and the 1961 labour value of the numeraire commodity (agricultural output). Since the price data are expressed in thousands of 1971 Canadian dollars and employment data in person-years, the estimated value of this intercept (4.4160) means that $4,416 of Canadian agricultural products, measured in constant 1971 prices, were produced for the 1961 final demand per person-year of labour employed. Therefore, the estimated vertical intercept provides a measure of the total labour productivity level in the numeraire commodity-producing industry (agriculture), i.e. the maximum real wage in constant dollar units. The estimated horizontal intercept of 169.07 indicates the maximum rate of profit in percent which is compatible with the 1961 technology. The maximum rate of profit is simply the reciprocal of the maximum eigenvalue of the 1961 matrix of integrated material requirements and thus determined by the material intensity of production in the initial year of observation. Comparisons of the 1961 wage-profit function with the estimated $w - r$ relations for the other years of observation can reveal the general properties of technological change, because such comparisons are independent of the numeraire chosen for the price system. (Pasinetti, 1977, p.159)

Observe that the estimated maximum real wage increases steadily over time from 4.4160 in 1961 to 9.2394 in 1980. The growth of the maximum real wage is consistent with the *labour-saving* tendency of technological change over the entire time period investigated. Thus Marx's first hypothesis, which relates to full automation, gains support from the upward trend of the maximum real wage.

Note that the maximum rate of profit declines steadily from 169.07% in 1961 to 150.75% in 1980. This result implies that technological change was *material-using* over the entire 1961–80 time period. Hence, Marx's second hypothesis is also not contradicted by our empirical findings.

Last, but not least, the shift pattern of the successive intersections of the wage-profit curves appears to be consistent with Marx's third proposition which predicts the falling tendency of the rate of profit. Observe that the profit rate at the switchpoints decreases steadily from 150% (1961–1966) to 10% (1976–1980).

Whereas the 1966 technology is cost-reducing relative to the previous technology in use at any rate of profit up to 150%, the 1980 technology could be adopted by a profit-maximizing system only if the actual rate of profit were below 10%. This suggests strongly the possibility of the falling rate of profit over the period investigated.

CONCLUSIONS

This study tests Marx's three hypothetical propositions concerning the nature of capitalist development in the very long-run. It is demonstrated that the three long-period tendencies (full automation, environmental destruction, and falling rate of profit) can coexist within the logical framework of the contemporary Leontief-Marx-Sraffa production model, provided that Marx's predictions are viewed as conditional statements rather than absolute truths. Although Marx's scenario for an historical evolution of the capitalist mode of production does not possess general theoretical validity, it may obtain in specific empirical cases and therefore cannot be dismissed *a priori*. This finding constitutes the first principal conclusion of our paper.

A confrontation of Marx's hypotheses with an empirical application of the contemporary production model to Canadian input-output data of the 1961–80 time period provides the basis for the second major conclusion of this study. The long-period tendencies of technological

change in Canada seem to conform to Marx's forecast of capitalist development *à la longue*. This conclusion implies that Marx's economics is empirically relevant in at least one special case.

APPENDIX

The Canadian input-output tables and the associated labour inputs for 1961, 1966, 1971, 1976 and 1980 were obtained from the Structural Analysis Division of Statistics Canada in machine readable form. The 1961 and 1966 tables are in 1961 constant prices. The 1971 table was available in both 1961 and 1971 prices. The 1976 and 1980 tables are in constant 1971 prices. All price data are expressed in thousands of Canadian dollars. Labour inputs are expressed in jobs in any given industry and year. The aggregation level includes 43 industries. The accounting framework of the Canadian input-output data is described in detail in Statistics Canada (1983) and Statistics Canada (1984).

A modified version of the open model specified in Statistics Canada (1984, p.29) was chosen for the purposes of this study from among the several variants of Statistics Canada's input-output models. Basically, this version is a domestic supply base model in which domestic production satisfies total demand by industries and final demand categories (personal expenditure on goods and services + fixed capital formation + value of net physical change in inventories + gross government current expenditure on goods and services + exports – imports – government production). The final version was obtained through the following sequence of operations:

a) The 1961 and 1966 input-output tables were reconstructed to the 1971 constant price base with the aid of the deflators obtained from the 1971 tables which were available in both 1961 and 1971 prices.
b) Twelve service industries (#32–43) were eliminated to obtain for every year of observation a 31 × 31 matrix of input-output coefficients representing the basic commodity producing sector of the economy.
c) Employment in the service industries was eliminated from the labour data to obtain a 1 × 31 vector of direct labour input coefficients for every year of observation.

FOOTNOTES

1. For a detailed discussion of classical analysis see e.g. Walsh and Gram (1980).
2. This conceptual link between the matrix of capital stock coefficients and the Leontief technology matrix was originally introduced by Özol (1980).
3. Observe that (3) can be interpreted as a solution to the transformation problem.
4. For a brief description including proofs of Frobenius-Perron theorems see e.g. Pasinetti (1977, pp. 267–77).
5. The same result is derived for a two-departmental model in Morishima (1973, pp. 142–4).
6. The empirical evidence presented in Mandel (1975, chapter 5) and Perlo (1973, chapter 3) indicates that the rate of surplus value increases over time in capitalist economies.
7. Gillman (1957, chapters 4 and 5), Mandel (1968, chapter 5), and Mandel (1975, chapter 5) provide empirical evidence suggesting that the tendency of the falling rate of profit prevailed in some countries in particular periods.

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