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Random Cost Functions and Production Decisions

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INTRODUCTION

Various researchers on the economics of uncertainty have come up with models to explain the "overall" and "marginal" impacts of uncertainty on the level of output and input utilization of the firm. Some of these researchers obtain their results under the assumption that the only source of uncertainty is output price (e.g., Sandmo [1971], Ishi [1977], Batra and Ullah [1974], Hartman [1975], Katz [1983, 1985], Fooladi [1986]). Others assume input prices are random while output price is given (e.g., Okuguchi [1977], Blair [1974], Perrakis [1980], Stewart [1978]). Fooladi [1985] explores the effect of uncertainty, under the assumption that all prices (input and output) are random. Many economists also compare various economic decisions made by the firm under uncertainty with their counterpart decisions under certainty, where the sources of uncertainties are something other than the output or input prices. These sources are quite diverse and range from uncertain deliveries of inputs (Martin [1981]) and random flow of factor services (Ratti and Ullah [1983]) to some specific technological uncertainty (Macminn and Holtmann [1983]) or uncertainty in parameters of production function (Feldstein [1971]).

None of these researchers examine the effect of an uncertain cost function on the optimal level of output and input utilization of the firm. This effect can readily be determined with the framework established in this paper. The significance of assuming a random cost function is in its ability to summarize various types of uncertainties into one random variable. For instance, many of the above-mentioned sources of uncertainties (either solely or in combination) may create randomness in cost function.

THE MODEL

Random Cost Function with Certain Output Price

Consider a competitive firm seeking to maximize the expected utility of profits, $E([u(\pi)]$, in which $\pi = p \cdot x - c(x, \mu)$, where π is the profit, p is the output price, x is the output level, and $c(x, \mu)$ is the random cost function. The random element in cost function, μ, could be generated by an uncertain technological relation between output and input factors (i.e, an uncertain production function), uncertain deliveries of inputs, random flow of factor services, or any combination of these sources of uncertainties. For any given level of output, $c(x, \mu)$ depends upon μ which has cumulative distribution function $F(\mu)$.

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Assume

$$\frac{\partial c(x,\mu)}{\partial x} > 0, \quad \frac{\partial c(x,\mu)}{\partial \mu} > 0, \quad \text{and} \quad \frac{\partial^2 c(x,\mu)}{\partial x \partial \mu} > 0.$$

The last assumption is equivalent to the condition that, for all x, the sign of the partial derivative of marginal cost with respect to μ is positive, i.e., it is equivalent to $\partial MC(x,\mu)/\partial\mu > 0$. Alternatively, this assumption can be rewritten as $\partial/\partial x(\partial c(x,\mu)/\partial\mu) > 0$ which is equivalent to the condition that, as total expected cost increases (as a result of an increase in x), the dispersion of total cost will increase. This is analogous to the "principle of increasing uncertainty" (PIU) introduced by Leland [1972]. (Hereafter, this assumption will be referred to as PIU.)

It is assumed that that the firm is a price taker (p is given), that the decision concerning the volume of output must be made prior to the knowledge of the random parameter μ, and that the firm exhibits a strictly concave, continuous and twice differentiable utility function of profits, i.e. $u'(\pi) > 0, u''(\pi) < 0$.

The objective of the firm is

(1) $$\max_x E[u(\pi)] = E[u(p \cdot x - c(x,\mu))].$$

The first and second order conditions for the maximization are

(2) $$E[u'(\pi)(p - MC(x,\mu))] = 0 \quad \text{at } x^*$$

and

(3) $$E[u''(\pi)(p - MC)^2 - \frac{\partial MC}{\partial x}\mu'(\pi)] < 0 \quad \text{at } x^*$$

where $MC(x,\mu) = \partial c(x,\mu)/\partial x$ and x^* is the optimum level of output.

It can be easily seen from (3) that increasing marginal cost (for any given μ) is sufficient, but not necessary, to satisfy the second-order condition.

Having set forth the basic assumptions of the model and having derived the optimality conditions, it is now possible to examine the overall impact of uncertainty on the optimal levels of output and the quantities of inputs. More specifically, the optimal output and input demands under random cost function (as defined above) may now be compared with their counterparts determined when the cost function is known with certainty to be equal to its expected value, by the following theorem.

Theorem 1 *Under uncertainty, where the cost function is random for any given x, the risk averse competitive firm produces lower output than in the case where the cost function is known with certainty to be equal to $EC(x,\mu)$.*

Proof

(i) Following Leland [1972], define $\mu_i(x)$ such that $E[MC(x,\mu)] = MC(x,\mu_i)$.

(ii) Given the PIU and $u''(\pi) < 0$, we will have the following: $\mu \gtreqless \mu_i$ implies $MC(x,\mu) \gtreqless MC(x,\mu_i)$ and $u'(\pi) \gtreqless u'(\pi_i)$ where π_i is the level of profit (for given x) evaluated at μ_i.

(iii) From (ii) it follows that

(4) $$[u'(\pi) - u'(\pi_i)](MC(x,\mu) - MC(x,\mu_i)] \geq 0$$

for all μ, and

(5) $$E[u'(\pi) - u'(\pi_i)][MC(x,\mu) - MC(x,\mu_i)] = E[u'(\pi)MC(x,\mu)] - MC(x,\mu_i)E[u'(\pi)] > 0$$

(iv) From the first-order condition (2), we have

(6) $$E[u'(\pi)MC(x^*,\mu)] = pE[u'(\pi)].$$

Substituting (6) into (5) results in

(7) $$pE(u'(\pi)] - MC(x^*,\mu_i)E[u'(\pi)] = [p - MC(x^*,\mu_i)]E(u'(\pi)] > 0.$$

Therefore, because $E[u'(\pi)] > 0$, we have

(8) $$p > MC(x^*,\mu_i) = E[MC(x^*,\mu)]$$

which means that the optimal level of output under uncertainty will be decided at a point where the output price is greater than the expected marginal cost.

(v) When the cost function is given to the firm as $C(x,\mu_i)$ with certainty, the profit function is non-random. Therefore, maximizing the profit also maximizes the expected utility of profit. Thus, given a certain cost function, the firm will select $x = x_c$ such as to satisfy the following first and second order conditions:

(9) $$\frac{\partial\pi}{\partial x} = p - MC(x,\mu_i) = 0 \quad \text{at } x_c, \quad \text{i.e.,} \quad p = MC(x,\mu_i) = E[MC(x,\mu)] \text{ at } x_c$$

(10) $$\frac{\partial^2\pi}{\partial x^2} = \frac{-\partial MC(x,\mu_i)}{\partial x} < 0 \quad \text{at} \quad x_c, \quad \text{i.e.,} \frac{\partial MC(x,\mu_i)}{\partial x} > 0.$$

(vi) Comparing (8) and (9) clearly illustrates that, given increasing marginal cost with respect to x (for any μ), $x^* < x_c$.■

The implication of smaller output resulting from the presence of uncertainty is the lower usage of all inputs that are not inferior. This means that, in a two factor model, firm's demand for at least one of the inputs must decrease in order to decrease the output level and, if the production is so-called "well-behaved" (i.e., if $f_{12} > 0$), demand for all inputs decreases.

Random Cost Function and Output Price

The results of the previous section hold, even if the assumption of the given output price is relaxed, i.e., if it is assumed that the decision concerning the volume of output must be made prior to the knowledge of the market price.

To see this, consider the same firm and assumptions as above, except now assume that the output price is a random variable having subjective cumulative probability distribution $F(p)$, with the expected value $E(p) = \bar{p}$ and variance $\text{var}(p) = \sigma^2$. In this case, the objective function of the firm, the first and second order conditions for maximum remain the same (as (1), (2), and (3), respectively.) However, equation (6) can be rewritten as follows:

(6') $$\bar{p}E[u'(\pi)] + \text{cov}[u'(\pi),p] = E[MC(x,\mu)]E[u'(\pi)] + \text{cov}[u'(\pi),MC(x,\mu)]$$

Rearranging (6') and dividing through by $E[u'(\pi)]$, we will get

(11) $$\bar{p} - E(MC) = \frac{\text{cov}(u'(\pi),MC]}{E[u'(\pi)]} - \frac{\text{cov}[u'(\pi),p]}{E[u'(\pi)]}.$$

We know that

$$\frac{\partial u'(\pi)}{\partial\mu} = u''(\pi)\frac{\partial\pi}{\partial\mu} = u''(\pi)\left(-\frac{\partial C(x,\mu)}{\partial\mu}\right) > 0 \quad \text{and} \quad \frac{\partial MC}{\partial\mu} > 0,$$

by assumption. As a result, $\text{cov}(u',MC) > 0$. We also know that

$$\frac{\partial u'(\pi)}{\partial p} = u''(\pi)(x) < 0,$$

and hence, $\text{cov}(u',p) < 0$. This means that the right-hand side of (11) is positive, resulting in

(8') $$\bar{p} > E(MC) \quad \text{at} \quad x^*$$

which is the same condition as (8) and therefore the same conclusion may be drawn as in Theorem 1. To elaborate on this point, note that if the output price and the cost function are given to the firm with certainty, as $\bar{p}$ and $C(x, \mu_1)$ respectively, the profit function becomes non-random. Therefore, the firm will select $x = x_c$ such as to satisfy

$$\bar{p} = MC(x, \mu_i) = E[MC(x, \mu)] \quad \text{at} \quad x_c. \tag{9'}$$

Given increasing marginal cost with respect to x (for any μ), comparing (8′) and (9′) shows that $x^* < x_c$. Therefore, Theorem 1 can be restated as follows:

Theorem 2 *Under uncertainty, where the output price and the cost function are random, the risk averse competitive firm produces lower output than in the case where the output price and the cost function are known with certainty.*

Our argument regarding the utilization of input factors would be exactly the same as it was for the case of given output price and we will obtain the same conclusion as was reached above.

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