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Production of Education: Are Socioeconomic Characteristics Important Factors?

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In a review of 114 studies on public education, Hanushek (1979) finds that only a small proportion find any significant relationship between school resources and student performance while a large proportion of these studies find significant relationships between socioeconomic characteristics of communities (SEC hereafter) and educational outcomes. However most studies include SEC on the demand side as taste variables but not as productive factors.[1] If student background and socioeconomic characteristics of communities are important determinants of student performance, then communities with large amounts of characteristics that enhance (decrease) educational outcomes will spend less (more) on school resources to obtain the same level and quality of educational attainment than communities without such characteristics. Research results of the importance of SEC have policy implications regarding the current debate over how best to improve the performance of the educational system in the United States.

The importance of SEC in the production of local public goods has not received a systematic treatment in empirical research on education production. Some researchers have included measures of SEC only as scale variables without theoretical justification (Margo: 1986, Summers and Wolfe: 1977). However, Hamilton (1983), Oates (1981), and Hanushek (1979) have argued theoretically that SEC affects both the demand for, as well as the production of, local public goods and should be included in the estimation of production functions of local public goods. Dynarksi, Schwab and Zampelli (1989), using a production function approach and data from California school districts, finds SEC to be important determinants of educational outcomes. Other studies (Baum: 1986, Gyimah-Brempong: 1989) have justified the inclusion of SEC in the production function of public goods.

These studies have not, however, treated education as a joint production process. The same set of inputs are combined to produce many educational outputs. Moreover, the relationships among these educational outputs may be noncausal and, hence, may not be correctly modeled with a simultaneous equation model. It is, therefore, necessary to use an approach that accounts for jointness in the production of education without assuming causal relationships among the educational outputs.

Researchers who have included SEC in their studies of education production functions have employed restrictive functional forms and have based their conclusions about the importance of SEC solely on *t* tests.[2] Those who have employed flexible functional forms to investigate education production (Cohn, Rhine and Santos: 1989, Callan and Santere: 1989, and Jimenez: 1986), on the other hand, have not investigated the effects of SEC on education production.

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This paper uses data from local school districts in the state of Michigan and a multiproduct translog cost function to investigate the importance of SEC in the production of education. This paper, therefore, extends the research on education production in this area using a different methodology---the translog cost function---and a different sample. We use three variables to proxy SEC. The use of the translog function allows us to account for the jointness of education production, a fact that previous research on education production has ignored.

The approach employed in this paper is straightforward: we estimate two multiproduct translog cost functions---with and without SEC as inputs---and then use a likelihood ratio test to test the hypothesis that the two estimated equations are not significantly different against the alternative that they are. The three variables used to proxy SEC are likely to be correlated. Therefore, having established the importance of SEC in the production of education, we search for which of the SEC variables or a combination thereof will be an adequate representation of SEC in an education production function in order to reduce collinearity problems. We then discuss some implications of excluding SEC from education production functions.

Our approach to the investigation of the importance of SEC in the production of education is different from those of previous researchers in some ways. First, we use a flexible functional form that imposes few restrictions on the structure of the underlying education production function. Second, we account for jointness in the production of education. Third, we use a likelihood ratio test rather than the *t* test to investigate the importance of SEC in the production of education. Finally, we introduce SEC into our estimated equation based on theoretical reasoning rather than as a scale variable. To our knowledge, this is the only study that employs the multiproduct translog cost function to investigate the effects of SEC on the production of education.

Understanding the importance of SEC in the production of local public goods is potentially very important, especially in an era of severe fiscal pressures on local governments. Given the limited capacity of local governments to improve the delivery of local services through increased public expenditures, they are likely to turn to other policy instruments at their disposal to increase the bundle of benefitial SEC as a means of improving the performance of local public schools. Second, if SEC is an important determinant of education output, then the debate over improving the quality of schools should not be limited to increasing the quantity and quality of school resources, but should be extended to include efforts to improve the home and environmental conditions of the student.

The rest of the paper is organized as follows: Section II specifies the econometric model, section III describes the data, section IV presents the empirical results, and section V concludes the paper.

THE MODEL

The theoretical foundation of this paper is provided by the works of Oates (1981), Hamilton (1983), and Bradford, Malt and Oates (1969). In the production of local public goods, Oates (1981) makes a conceptual distinction between two types of outputs---the services directly produced by the public agency (**D** output) and the outputs that are of concern to the citizen-voter (**Q** output).[3] **Q** output is the output that enters the citizen-voter's utility function while **D** output is the output that is directly produced with purchased inputs (eg. number of hours of instruction in mathematics). The production of **D** output depends solely on purchased inputs while the output of **Q** output depends on the quantity of **D** output and socioeconomic characteristics of the community. Though the citizen-voter pays taxes to purchase inputs for the production of **D** output, he is primarily concerned with **Q** output when he votes on matters relating to the financing of schools. Let **X** represent a vector of purchased inputs. The output of **D** output depends on **X**, and **Q** output uses **D** output and SEC as inputs.

Employing the human capital approach, we might argue that education provides for the development of certain skills on the part of students; hence, we may interpret the vector **Q** as measuring the degree of achievement of this objective. The quantity of **Q** output is determined in part by **D** output and a number of other variables that describe the "environment" in which **Q** outputs are produced (Margo: 1986, Summers and Wolfe: 1977, Hanushek: 1979, Dynarski, Schwab and Zampelli: 1989, Baum: 1986, Boardman, Davis and Sunday: 1977).

From the foregoing discussion, **D** is related to **X** in the following manner:

$$D = g(X) \qquad (1)$$

where all variables are as defined in the text above. **Q** output in turn is a function of **D** and **Z**, implying that in its reduced form, **Q** is a function of **X** and **Z**. **Z** might describe home backgrounds of the pupils such as education of parents, as well as neighborhood conditions such as level of safety. **Z** can be considered a vector of fixed inputs for each community at any time but vary across communities. We assume that a concave twice differentiable **Q** production function exists. The relationship between **Q** on the one hand, and **X** and **Z** on the other identifies the average production relationship. Individual producers (School districts) may use more or less inputs than the average in producing a given output. The difference between average input use and that used by a particular producer is assumed known to that producer but not to the researcher. In view of this, we specify the production function of education as an additive general error model (AGEM) production function (McElroy: 1987).[4] Formally:

$$Q_i = Q(X, Z - \varepsilon), \qquad X_i - \varepsilon_i \geq 0 \quad Qx \geq 0 \qquad (2)$$

where **Q** is a measure of educational achievement and ε is a vector of error terms assumed to have a zero mean and a positive definite covariance matrix. It is the relationship between **Z** and **Q** output that is of interest to us in this paper.

If **Z** takes the form that enhances (decreases) the productivity of purchased inputs, then communities with large quantities of **Z** will obtain higher (lower) quality of education output as other communities with similar levels of **X** but lower levels of **Z**. An alternative way to put this idea is that communities with higher levels of productivity enhancing (decreasing) **Z** will have to use less (more) purchased inputs than communities with lower levels of **Z** to obtain the same quantity and quality of education. **Z** is similar to any fixed factor of production in a neoclassical production function. Equation (2) should, therefore, be considered as a short-run production function.

The introduction of SEC into the educational production function creates problems for the interpretation of coefficients because it becomes almost impossible to disentangle the effects of purchased inputs on educational outcomes from those of SEC. To illustrate this point, suppose that Q is a function of X and Z as indicated in (2) and that X itself is a function of Z (as it is likely to be) and other variables (A); i.e. $X = X(Z,A)$. The last equation, together with equation (2) implies that $Q = Q(X(Z,A),Z)$. From this equation, $dQ/dX = \partial Q/\partial X(\partial X/\partial Z + 1)$; a combination of the effects of purchased inputs on educational outcomes and the effects of Z on X. To circumvent this problem, we use a cost function approach in our investigation.

The operating assumption in this paper is that local School Boards maximize the output of **Q** subject to a budget constraint imposed by the political process. We also assume that local School Boards operate in competitive input markets. The dual of output maximization subject to a cost constraint is cost minimization subject to an output constraint (Fuss and McFadden: 1978). Assuming that education output is maximized subject to a budget constraint, the result will be the same as minimizing the cost associated with the production of the optimal output. We can, therefore, specify and estimate a cost function and derive the characteristics of the underlying production function from the estimated cost function. The cost function has input prices (W), Q, and **Z** as arguments. McElroy (1987) has shown that the AGEM cost function associated with the AGEM production function is:

$$\begin{aligned} C(.) &= C(W,Q,Z,\varepsilon) \\ &= C(W,Q,Z) + \sum w_i \varepsilon_j, \qquad C_w, C_q > 0 \quad C_z \leq 0, \text{ or } \quad C_z \geq 0 \qquad (3) \end{aligned}$$

where C_i is the first derivative of cost with respect to *i*th argument, $i = W,Q,Z$, C(.) is the cost function, and $\sum w_j \varepsilon_j$ s input price weighted sum of error terms. We note that this cost function is a variable cost function with Z as quantities of fixed nonpurchased inputs. Since Z is a vector of nonpurchased inputs which increases (decreases) output without increasing purchased inputs, the cost function is decreasing (increasing) in **Z** if **Z** take forms that increases (decreases) output.

To estimate the model econometrically, we need to provide a specific functional form for the AGEM cost function we have specified. We use the multiproduct translog cost function (Christensen, Jorgenson and Lau: 1973) to represent the AGEM cost function. The translog cost function imposes very few restrictions on the structure of the underlying production function, allowing us to treat restrictions on the production function as testable hypotheses. It can also model other functional forms, such as the Cobb-Douglas as a special case.[5] It also allows us to take account of jointness in education production. Any well behaved cost function must be linearly homogenous in input prices and the second order coefficients must be symmetric. Therefore, we impose linear homogeniety and symmetry restrictions on the cost function.

Three outputs, four purchased inputs, and three nonpurchased inputs are the arguments of the cost function. The multiproduct translog cost function with linear homogeneity and symmetry restrictions imposed is given as:[6]

$$\begin{aligned}\ln C = \alpha_0 &+ \sum_i \alpha_i \ln Q_i + \sum_j \beta_j \ln W_j + \frac{1}{2}\sum_i\sum_k \alpha_{ik} \ln Q_i \ln Q_k + \frac{1}{2}\sum_j\sum_l \beta_{jl} \ln W_j \ln W_l \\ &+ \sum_i\sum_j \delta_{ij} \ln Q_j \ln W_j + \sum_i \gamma_n \ln Z_n + \frac{1}{2}\sum_n\sum_m \gamma_{nm} \ln Z_n \ln Z_m \qquad (4a) \\ &+ \sum_i\sum_n \varepsilon_{in} \ln Q_i \ln Z_n + \sum_j\sum_n f_{jn} \ln W_j \ln Z_n + \varepsilon_i\end{aligned}$$

$$\sum_j \beta_j = 1,\ \sum_j \beta_{jl} = \sum_j \delta_{ij} = \sum_j f_{jn} = 0,\ \beta_{jl} = \beta_{lj}, \quad \alpha_{ik} = \alpha_{ki}, \quad \gamma_{ij} = \gamma_{ji}$$

where α_i, β_j, α_{ik}, β_{jl}, γ_{ij}, δ_n, δ_{nn}, ε_{in}, and f_{jn} are parameters to be estimated, ε_i s a stochastic error term, and all other variables are as defined above. Setting all second-order coefficients to zero produces the Cobb-Douglas functional form. From Shepherd's Lemma, the input share equations are given as:

$$S_j = \beta_j + \sum_i \gamma_{ij} \ln Q_i + \sum_l \beta_{jl} \ln W_l + \sum_n f_n \ln Z_n \qquad (4b)$$

for $j = 1,2,3,4$ where $S_j = \partial \ln C / \partial \ln W_j$ is the share of input j in total cost. Equation (4a) can be estimated alone; so can equation (4b). However, equation (4b) contains structural information that will improve the efficiency of estimation of the cost function. Because of symmetry, estimating equations (4a) and (4b) as a system improves estimation efficiency without adding to the cost of estimation.

Our objective in this paper is to find out whether SEC are important factors in the production of education. If SEC are not important in the production of education, then an education cost function with **Z** as added inputs will be the same as one without **Z**. Restricting the coefficients of **Z** in equations (4a) and (4b) to zero, we obtain another system of cost and input share equations for the production of education without SEC as inputs. We refer to these set of equations as the truncated model.

If **Z** significantly decreases (increases) the cost of producing education, the two systems of equations will be different; otherwise they will be the same. We, therefore, estimate and compare the two sets of equations.

DATA AND ESTIMATION PROCEDURE

A cross section data of 175 Michigan school districts with populations of 1,000 or more for 1985 is chosen for this study. This sample was chosen in part because of availability of unusually rich data set for that year. We also limited the sample to school districts with 1000 or more people because one of our outputs is High school ACT scores. School districts with very small populations may not have a high school. Since we estimate the equations in log forms, observations from such school districts would be excluded anyway.

The cost of production is measured as the total expenditure on education excluding transportation in each school district. Four purchased inputs---teachers (INST), instructional support services (SUPINST), noninstructional support services (SUPNIST), and capital (CAPT)---are used in estimating the cost function. It is usual to average teacher salary as the price of instruction. However, given Summers and Wolfe's (1977) finding that teacher quality has a substantial impact on student performance, we adjust average teacher salary for quality.[7] We assume that teacher quality is directly related to highest degree attained by the teacher and teacher experience. The quality adjusted cost of instruction was calculated as follows: We took the product of degree and experience, mean-scaled this variable to obtain a teacher quality index, and divided the average teacher salary by this quality index. This means that for two districts with the same average salary, the district with fewer years of experience or lesser educational attainment of teachers or both will have a higher quality adjusted teacher salary.

The data for teachers salary included fringe benefits while those of support staff did not. To make the other two labor prices comparable, we adjusted the salary of support staff upwards by 20%. All these input prices are measured in $ per annum. Assuming that school districts are in long-run equilibrium, the relevant price of capital is the user "cost" of capital. However, we could not get any meaningful school district level data to calculate the user cost of capital. Therefore, we took the total annual per pupil expenditure on capital as the cost of capital. We recognize that this is not a perfect measure of capital cost; however, it may be better than excluding capital from the cost function given data limitations.[8] The data on costs and input prices were obtained from Michigan State Board of Education, *Michigan K-12 School Districts Ranked by Selected Financial Data*, 1985/86 (East Lansing, Michigan).

Three educational outcomes---test scores in english (READ) and mathematics (MATH) from the Michigan Educational Assessment Program (MEAP), and full time equivalent enrolment in a school district (ENROL)---are used as the measures of output. The MEAP program is designed to test the achievement of 10th grade students; hence, it is taken by all 10th grade students in the state of Michigan.[9] While some researchers use enrollment as the measure of the school system's output (Jimmenez: 1986, Callan and Santerre: 1989), others (Hanushek: 1979, Margo: 1986) use test scores as the measure of educational output. While neither measure may be a perfect measure of school output, a combination of the two measures may provide a more complete measure of educational output than either measure by itself. Hanushek (1979) argues that most evaluations of a school system's performance are based on test scores which both students and parents judge to measure the quality of schools. It can be argued that the number of students educated by a school system also constitute an output of the school district. The two types of output we employ reflect the *quantitative* and the *qualitative* aspects of the school district's output, with enrolment measuring the quantitative dimension while test scores measure the qualitative dimension.

Data for ENROL, READ, and MATH were obtained from Michigan Department of Education, *Intermediate School District Report, 1985/86*, (East Lansing, Michigan: 1987).

Several variables can be used to represent the socioeconomic characteristics of a community. Median family income (INC) readily comes to mind. Hamilton (1983) has argued that income summarizes some of the most important aspects of SEC in a community. Income influences achievement in many ways. For example, wealthy parents can and do provide additional reading material and tutoring for their children at home. These children can, therefore, be educated to a given level with fewer school resources.

The educational attainment of parents affects a student's academic performance. Educated parents are more likely to help their children on homeworks, provide extra reading material, and in general, complement the work of teachers than uneducated parents. We, therefore, use educational attainment of the adult population, measured as the percentage of the adult population with the equivalent of high school education or more (EDHS), as our second measure of SEC. Finally, a safety index (SAFE), measured as the inverse of FBI index crime rate, is included to capture the idea that a safer school environment is more conducive to learning than a crime ridden environment.

Some authors use the racial composition of communities as a measure of SEC and find it to be a significant factor in the production of local public goods. We believe, however, that race by itself is not an important variable in the production of local public goods. The supposed "racial differences" in attitudes can be explained by differences in other socioeconomic characteristics (Gyimah-Brempong: 1986). We, therefore, do not use race as a variable representing SEC. Income, education and safety are beneficial socioeconomic characteristics which are expected to enhance the output of the school system. Since these are nonpurchased inputs, we expect them to have negative coefficients in the education cost function.

Data on INC and EDHS were obtained from Michigan Department of Commerce, *Business Information 1985*, (East Lansing, Michigan). The safety index data were obtained from Michigan Department of State Police, *Crime in Michigan: Uniform Crime Report*, 1985 (East Lansing, Michigan). Table I presents summary statistics of the data. The sample shows wide variations in the variables, reflecting differences in educational achievement, expenditures on schools, and community characteristics. We mean scale the variables for estimation purposes.

Table I

Summary Statistics and Data

Variable	Mean	Standard Error
Total Cost	20,811,212.19	57,979,255.00
Enrollment	5,979.09	16,647.35
ACTE	83.20	7.74
MATH	70.28	11.06
DEGREE	1.63	.18
INST	29,729.18	4,170.61
EXPER	15.29	3.72
SUPIN	20,124.93	4,820.70
SUPNI	26,400.04	5,711.94
CAP	190.14	139.31
INC	21,834.21	6,276.52
SAFE	0.11	0.764
EDHS	69.24	9.56
Total Cost Share of Instruction	.5354	.0375
Total Cost Share of Instructional Support	.0814	.0332
Total Cost Share of Noninstitutional Support	.3121	.0318
Total Cost Share and Capital	.0711	.0302

N = 175

The cost of capital as measured here may be subject to measurement error and, therefore, could introduce bias to the coefficient estimates. An alternative to solving this potential problem is to exclude the price of capital from the cost function. This could also introduce specification bias. To resolve this dilemma, we used a likelihood ratio test to test whether we could exclude the price of capital from the cost function without misspecifying the cost function. The calculated $/^2$ statistic of 29.75 (Table III) implies that one cannot exclude the price of capital from the translog cost function without specification error. We, therefore, include the price of capital in the cost function while noting the possible measurement error.

It is possible that some arguments in the cost function (such as output and input prices) are endegenous. In that case, a full information maximum likelihood estimation procedure is the appropriate estimating methodology. Using Hausman's specification test (Hausman: 1978) to test for the endogeniety of all outputs and input prices, we are unable to reject the null hypotheses of exogeniety of outputs and input prices at = .10.[10]

The cost functions are estimated jointly with the cost share equations. Since the cost shares add up to unity, we deleted one share equation---capital share---in estimating the system of equations to avoid perfect collinearity. Zellner's (Zellner: 1962) iterative seemingly unrelated regressions (ITSUR) procedure is used to estimate the system. This estimation procedure produces maximum likelihood coefficients and the estimates are invariant as to which share equation is deleted from the system of equations for estimation.

RESULTS

Coefficient estimates and their asymptotic standard errors for the translog cost function are presented in Table II. Column 2 presents coefficient estimates for the full model, column 3 presents estimates for the truncated model, while column 4 presents coefficient estimates for a Cobb-Douglas specification of the cost function. The translog cost function fits the data rather well, with adjusted R^2 being above .90 for each of the three specifications of the cost function.

Table II

Coefficient Estimates of Translog Cost Function

Parameter	Full Model	Model Without SEC	Cobb Douglas Specification
α_0	0-.1692.	0345	-.0914
	(2.38)*	(2.17)	(1.79)
α_1	.7946	.9262	.7882
	(14.67)	(26.99)	(20.38)
α_2	.4249	-.0883	.0909
	(1.72)	(0.70)	(0.26)
α_3	.3045	.1078	.3504
	(2.15)	(0.61)	(1.90)
α_{11}	.0646	.0694	------
	(1.35)	(1.33)	
α_{12}	.5833	-.3826	-----
	(0.521	(0.69)	
α_{13}	-.1990	.4418	------
	(1.19)	(1.54)	
α_{22}	.7835	-.3866	------
	(0.86)	(0.07)	
α_{23}	-.66592	.9484	------
	(0.73)	(1.14)	
α_{33}	-.8362	-3.9288	------
	(0.40)	(2.86)	
β_1	.4651	.4574	.4592
	(28.46)	(120.60)	(155.11)
β_2	.0724	.0795	.0714
	(9.04	(19.20)	(20.11)
β_3	.2794	.2605	.2698
	(37.95)	(68.02)	(79.28)
β_4	.1831	.2026	.1998
	(15.36)	(34.40)	(40.79)
β_{11}	.0236	.0246	------
	(8.18)	(4.39)	

Parameter	Full Model	Model Without SEC	Cobb Douglas Specification
β_{12}	.0002	.0005	------
	(1.58)	(0.11)	
β_{13}	-.0291	-.0304	------
	(5.30)	(5.54)	
β_{14}	.0053	.0050	------
	(1.50)	(1.68)	
β_{22}	.0118	.0129	------
	(2.08)	(2.21)	
β_{23}	-.0126	-.0142	------
	(2.48)	(2.78)	
β_{24}	.0006	.0008	------
	(1.55)	(0.48)	
β_{33}	.0465	.0493	------
	(5.58)	(5.86)	
β_{34}	-.0049	-.0050	------
	(1.95)	(1.60)	
β_{44}	-.0001	.0008	------
	(1.70)	(1.43)	
γ_1	-.1546	------	.4681
	(2.21)	(4.18)	
γ_2	-0.0903	-------	.0432
	(1.79)	(2.37)	
γ_3	-.4589	-------	.2074
	(2.69)	(0.89)	
γ_{11}	.8110	------	------
	(1.10)		
γ_{12}	.1549	------	------
	(1.24)		
γ_{13}	-1.4743	------	------
	(1.10)		
γ_{22}	-.0145	------	------
	(0.69)		
γ_{23}	-.1279	------	------
	(0.56)		
γ_{33}	.7285	------	------
	(1.48)		
δ_{11}	-0.111	-.0149	------
	(1.82)	(2.45)	
δ_{12}	.0128	.0140	------
	(1.51)	(2.48)	
δ_{13}	.0060	-.0035	------
	(1.19)	(0.65)	
δ_{14}	-0.0095	.0045	------
	(1.35)	(0.97)	
δ_{21}	.0095	.0198	------
	(0.20)	(0.44)	
δ_{22}	-.0471	-.0361	------
	(0.856)	(0.68)	
δ_{23}	.0211	.0129	------
	(0.42)	(0.28)	
δ_{24}	.0164	.0034	------
	(0.80)	(0.61)	
δ_{31}	0.071	.0149	------
	(0.27)	(0.62)	
δ_{32}	.0315	.0271	------
	(1.08)	(0.97)	
δ_{33}	-.00440	.0329	------
	(0.11)	(1.28)	
δ_{34}	-.0001	-.0090	------
	(0.23)	(0.81)	
ε_{11}	-.1863	------	------
	(0.74)		
ε_{12}	-.0102	------	------
	(1.04)		
ε_{13}	.5183	------	------
	(0.99)		
ε_{21}	-1.3664	------	------
	(1.25)		
ε_{22}	-.2876	------	------
	(1.27)		
$_{23}$	2.5408	------	------
	(1.05)		
ε_{31}	2.4504	------	------
	(1.01)		
ε_{32}	.8629	------	------
	(2.02)		
ε_{33}	-5.8803	------	------
	(1.56)		
f_{11}	-.0056	------	------
	(0.38)		
f_{12}	-.0359	------	------
	(2.10)		
f_{13}	*-.2103*	------	------
	(0.66)		
f_{14}	.0517	------	------
	(1.33)		
f_{21}	.0031	------	------
	(1.28)		
f_{22}	-.0022	------	------
	(1.46)		
f_{23}	.0076	------	------
	(3.22)		

Parameter	Full Model	Model Without SEC	Cobb Douglas Specification
f_{24}	-.0075 (2.21)	------	------
f_{31}	.0204 (0.65)	------	------
f_{32}	.0592 (1.67)	------	------
f_{33}	-.0467 (1.45)	------	------
f_{34}	-.0328 (1.78)	------	------
$\bar{R}^2$	.9661	.9071	.9095

*Absolute values of asymptotic 't' statistic in parentheses.

+Outputs indexed as follows: 1 = ENROL
2 = ACTM
3 = ACTE

Inputs indexed as follows: 1 = INST
2 = CAPITCAL
3 = SUPPORT SERVICE

SEC indexed as follows: 1 = INC
2 = SAFE
3 = 3DHS

Before discussing the coefficient estimates, we test hypotheses about the translog cost function. Hypotheses test are based on the likelihood ratio statistic:

$$-2log\Lambda = N[log|\Omega_r| - log|\Omega_u|] \qquad (5)$$

where $|\Omega_r|$ and $|\Omega_u|$ are absolute values of the determinants of the estimated error covariance matrices for the restricted and unrestricted models respectively, and N is the number of observations. This statistic is distributed as chi-square with degrees of freedom equal to the number of parameter restrictions.

Statistics to test the various hypotheses about the translog cost function are presented in Table III. The likelihood ratio statistic to test the hypothesis that the education cost function is of the Cobb Douglas function form is 205.43 which is too far into the rejection region. This, together with the fact that there are sign reversals in some of the coefficients of the Cobb Douglas model leads us to reject the Cobb Douglas functional form. All our discussion will, therefore, be based on the translog specifications. The statistic to test the hypothesis of equality between the truncated and the full cost functions is 86.50, leading us to reject the null hypothesis of equality between the two cost functions.

All the first-order output coefficients in Table II are positive and significantly different from zero at = .10 or better. Most of the second order output coefficients are positive but are statistically insignificant. All the first order input price coefficients are positive and significantly different from zero at = .01. This implies that the cost function as well as the input cost share equations increase with input prices. Most of the second order input price coefficients in all three specifications are significantly different from zero at = .05 or better. The signs and magnitudes of these coefficients indicate that the data does not, at least, violate the cost minimization assumption.

Table III

Statistics for Hypothesis Testing

Hypothesis	Degree of Freedom	Critical Value *	Calculate Chi-Square
No Capital	10	23.21	29.75
SEC is not important	25	44.31	86.50
Cobb-Douglas Specification	45	69.96	205.43
INC is not important	10	23.209	53.60
SAFE is not important	10	23.209	72.81
EDHS is not important	10	23.209	58.31
SAFE and EDHS are not important	17	33.41	56.87
INC and EDHS are not important	17	33.41	48.56
INC and SAFE are not important	17	33.41	52.30

*All tests at = .01.

Importance of SEC in Education Cost Functions

In the full model, the first order coefficients of all the SEC variables are negative and significantly different from zero at = .10 or better. The second order coefficients of the SEC variables are mixed, with some coefficients significantly different from zero, and others not. With none of the direct second order coefficients significant, the negative and significant first order coefficients of the SEC variables imply that the education cost function is decreasing in the endowments of these SECs.

The coefficient estimates of the SEC variables give some indications that SEC are important nonpurchased inputs in the production of education and, hence, affect the cost of producing education. The negative and significant coefficients of these variables indicate that higher levels of safety, income and educational attainment of the adult population increases the output of education, given the quantity and quality of purchased inputs, hence, reducing the cost of producing any given level of education output. These coefficient estimates are consistent with the results obtained by Margo (1986), Summers and Wolfe (1977), Hanushek (1979), and Dynarski, Schwab and Zampelli (1989) among others.

To test the null hypothesis that socioeconomic characteristics of communities have no influence on the cost (production) function of education against the alternative that they are important factors, we compare the full cost function with the truncated model. Comparing columns 2 and 3 of Table II, we see that the coefficients of the truncated model are different in absolute magnitude from those of the full model. In addition to the negative coefficient of READ in the truncated model, the coefficient of MATH and READ in the truncated model are statistically insignificant. The negative coefficient of READ and the insignificant second order output coefficients implies a negative marginal cost of producing english reading and comprehension in Michigan schools---a finding that is inconsistent with production theory. Differences in coefficients, combined with the fact that the likelihood ratio test rejected the hypothesis of equality between the full and truncated models, leads us to concluded that the SEC variables are jointly important determinants of education production.

The three measures of SEC are likely to be correlated, hence, including all three in the same cost function could introduce collinearity problems.[11] To investigate whether it is possible to reduce the number of SEC variables in the cost function as a way of reducing collinearity, we estimate the cost function with different combinations of the three SEC variables and compare them with the full model. Since there are six possible combinations of the three SEC variables, there are six such cost functions.[12] With few exceptions, the coefficient estimates of these cost functions have similar signs as those of the full model although the absolute magnitude of the coefficients differ from those of the full cost function. From the statistics reported in Table III, we reject the null hypothesis of equality of each of these restricted cost functions and the full cost function.

We also tested the hypothesis of equality of cost functions with only one SEC variable and the truncated cost function. The χ^2 statistics are 22.26, 24.91, and 25.81 for equality of the cost functions with INC, SAFE, and EDHS as measures of SEC, respectively, and the truncated model. With 5 degrees of freedom each, we reject each these hypothesis at $\alpha = .01$.

Intuitively, all SEC variables are important determinants of educational outputs and should all be included in the education production (cost) functions. We expect the educational problems, hence the related costs, associated with different combinations of the SECs to be different. For example, the educational problems (and cost) in a low income, highly educated, and high crime community are likely to be different from those of a low income, highly educated, and low crime community. It is, therefore, desirable to include as many SEC measures in an education production (cost) function as is possible in order to reflect the many possible configurations of SECs that influence the outputs of education.

What is the practical significance of the result that SEC are important inputs in the production of education? We noted earlier that the flexibility of cost with respect to each of these SEC variables is negative. This implies that failure to include the SEC variables in a cost function will result in an overestimate of the cost of producing a given quantity and quality of education. One practical use of empirical studies of the education production (cost) function is to help answer the question whether cost reductions can be achieved by increasing or decreasing the scale of operation or whether it is possible to substitute among inputs in order to reduce production cost. To assess the importance of SEC in such policy questions, we calculate overall economies of scale (SCALE) from the full model and compare it to one calculated from the cost function without SEC as arguments. For the multiproduct firm, Panzar and Willig (1977) define and measure scale economies (SCALE) as:

$$scale = \Big(\sum_i \partial \ln C/\partial Q_i\Big)^{-1}$$

There are increasing returns to scale if $SCALE > 1$, constant returns to scale if $SCALE = 1$ and decreasing returns to scale if $SCALE < 1$.

The calculated SCALE from the full model is .6562 while the SCALE calculated from the truncated model is 1.0574. While the full cost function indicates that there are decreasing returns to scale in the production of education, the cost function without SEC as inputs indicates that there are constant returns to scale in the production of education at the means of the variables. It seems the truncated cost function overestimates scale economies in the production of education in Michigan. This overestimate is large enough to have a negative impact on a decision to expand the overall production of education in a school district. The overestimate of diseconomies of scale implied by the truncated cost function may favor a decision to break up large schools or work against the consolidation of schools. Failure to include SEC in the education production (cost) function may, in part, explain why large schools, which tend to be in poor inner city communities with lower endowments of positive SECs, tend to have a higher explicit cost of producing a "unit" of education.

Factor Substitution

An important aspect of production technology is the degree to which factors of production are substitutable in production. To provide further evidence of the differences between the full and truncated models, we estimate and compare Allen partial elasticities of factor substitution (AES) for the two models. These are presented in Table IV. Panel (a) presents the calculated AES from the full model while panel (b) presents the AES from the truncated model. The AES from both models show that all factors are substitutes. The calculated elasticities are significantly different from zero and from unity at any reasonable confidence level. The calculated AES provide another evidence that the production function is of neither the Leontief nor the Cobb Douglas functional form.

Table IV

Estimates of Elasticities of Factor Substitution*

A.

INST	-0.7854 (.1938)*	1.0046 (.3808)	.8259 (.2197)	1.1392 (.2025)
SUPINST		-9.5343 (.8638)	.5039 (.1664)	1.1036 (.5285)
SUPNIST			-1.7268 (.1425)	7792 (.2449)
CAP				-13.0721 (2.4820)

B.

INST	-.7819 (.4006)	1.0046 (.4308)	8259 (.2969)	1.1313 (.2993)
SUPINST		-9.3381 (.9934)	.8346 (2079)	1.1382 (.9891)
SUPNIST			-1.6981 (.1634)	.7747 (.2867)
CAP				-12.8942 (2.8691)

*asymptotic standard errors in parentheses.**Table I**

Summary Statistics of Data

Though the calculated AES from both models classify all factors as substitutes, there are slight differences in the absolute magnitudes of the elasticities of substitution from the two models. With the exception of the AES between noninstructional support services and instruction and between instructional support and instruction, the AES calculated from the full model are generally different from those calculated from the truncated model. A few of these differences are quite large in absolute magnitude. Though in absolute terms, the differences between the two sets of substitution elasticities tend to be generally small, in relative terms, these differences are large. The degree to which the truncated model overestimate the elasticities of substitution range from about -.6% for the AES between instructional support services and capital to a 65.6% for the AES between instructional support services and noninstructional support services. The differences in the two sets of calculated AES is another indication that the full and truncated models are different

The conclusion that socioeconomic characteristics of communities have significant impact on the cost (production) of education is consistent with the results of previous research (Summers and Wolfe: 1977, Margo: 1986, Hanushek: 1979, Dynarski, Schwab and Zampelli: 1989, Baum: 1986, Boardman, Davis, and Sanday: 1977, among others) that find a positive relationship between socioeconomic characteristics and educational outcomes.

While our results are similar to those of earlier researchers, our approach is different. Earlier researchers employed production function approach using mostly restrictive functional forms and one educational output in their analyses. We used a flexible cost function approach that included three SEC variables and considered education as a joint production process---an aspect of education production that has received very little attention in the literature. The use of the flexible cost function not only allows for the modeling of education as a joint production process, it also makes it possible to analyze other aspects of the education production technology, such as elasticities of factor substitution and economies of scale and of scope. It can also allow researchers to analyze how SEC can be "substituted" for purchased inputs in the production of education---analysis that is, at best, difficult to conduct with a production function approach.

The policy implication of this result is that, in addition to increasing school resources, it may be necessary to improve students' home environment as well as the larger socioeconomic environment of the community. Exclusive emphasis on increasing school resources may not be enough to improve the performance of schools in the production of education. As Hanushek (1981) has observed, society may be throwing money at the problem if there is an exclusive emphasis on increasing school resources as a means of improving educational outcomes. The research implication of our results is that exclusion of SEC from education cost (production) functions will result in misspecification of such functions. Results of research that exclude SEC from the cost (production) function of education should, therefore, be treated with caution.

CONCLUSION

This paper used data from school districts in the state of Michigan to investigate the importance of socioeconomic characteristics of local communities in the production of education using three measures of socioeconomic characteristics. We find that socioeconomic characteristics of communities are significant factors in the production of education. Therefore, excluding them from education production functions will result in misspecification and parameter estimates will be biased, hence policy conclusions from such parameter estimates will, at best, be questionable. Test results also indicate that all three measures of socioeconomic characteristics used in this study are important in the production of education. It is, therefore, important for researchers to include all three SEC variables in estimating education cost (production) functions.

NOTES

1.For more on the debate on the relative importance of school resources on the one hand and student background and socioeconomic characteristics of communities on the other, see Hanushek (1979, 1986).

2.For more on the various approaches to modeling education production, see Hanushek, (1979). Also see Dynarski, Schwarb and Zampelli (1989).

3.Oates (1981) uses the term **C** output for the output that is of concern to the citizen-voter. We use the term **Q** output to avoid confusing this output with the cost function which we employ later in the paper.

4.The AGEM production function models the error terms theoretically rather than append it to the production function after it has been modeled in a deterministic way as is the custom. McElroy (1987) shows that the dual of the AGEM production function is an AGEM cost function whose error terms are derived from the error terms of the underlying production function---a property other production (cost) functions do not have.

5.The translog function does have its own disadvantages. For example, it may not be self dual and may not be well behaved globally. However, regularity conditions exists within reasonable intervals to allow for efficient parameter estimates. In addition, it is more efficient than other flexible functional forms. For more on the comparisons of the translog function and other flexible functional forms, see Guikey *et al.* (1983).

6.Our specification of the translog cost function is very similar to the one specified and estimated by Cowing and Holtman (1983).

7.. We thank an anonymous referee of *This Journal* for motivating this measurement of the price of instruction.

8. See our discussion of the treatment of capital below.

9.Some authors have used average scores on college entrance tests such as the SAT and ACT as measures of school output. However, the average scores on these tests are sensitive to the proportion of students in a school district who take the test. The MEAP test does not have this problem since all students in a school district are *required* to take it.

10.The calculated χ^2 statistics for the test of the null hypotheses of exogeniety of ENROL, ACTE, ACTM, INST, SUPT, and CAPT are 2.83, 6.09, 3.89, 3.29, 5.42, and 3.09, respectively. With 15 degrees of freedom each, we cannot reject the null hypotheses. We used the SECs and school purchased school inputs as instruments in endogenizing the output and input prices in carrying out the Hausman test.

11.The Pearson correlation coefficients between INC and SAFE, INC and EDHS, and SAFE and EDHS are .49, .67 and .63, respectively. The correlation between INC and INST, SUPPORT, and CAP are .67, .81 and .41, respectively.

12.Because of space considerations, the coefficient estimates of these cost functions are reported. They are, however, available from the authors, upon request.

REFERENCES

1.Baum, D. N. (1986), "A Simultaneous Equations Model of the Demand for and Production of Local Public Services: The Case of Education," *Public Finance Quarterly*, 14 (2), pp. 157--178.

2.Boardman, A. E., O. A. Davis, and P. R. Sanday (1977), "A Simultaneous Equation Model of the Educational Process," *Journal of Public Economics*, 7, pp. 23--49.

3. Bradford, D. F., R. A. Malt, and W. E. Oates (1969), "The Rising Cost of Local Public Services: Some Evidence and Reflections," *National Tax Journal*, 22 (2), pp. 185--202.

4.Callan, S. and R. E. Santere (1989), "The Production Characteristics of Local Public Education: A Multi-Production and -Input Approach," Paper presented at the 15th annual convention of the Eastern Economics Association, March 3-5 1989.

5.Christensen, L. R., D. W. Jorgenson, and L. J. Lau (1973), "Transcendental Logarithmic Production Frontiers," *Review of Economics and Statistics*, 55, pp. 28--45.

6. Cohn, E., S. L. W. Rhine, and M. C. Santos (1989), "Institutions of Higher Education as Multi-Product Firms: Economies of Scale and Scope," *Review of Economics and Statistics*, 71 (2), May, pp. 284--290.

7. Cowing, T. G. and A. G. Holtmann (1983), "Multiproduct Short-Run Hospital Cost Functions: Empirical Evidence and Policy Implications from Cross-Section Data," *Southern Economic Journal*, 49, pp. 637--653.

8. Dynarski, M., R. Schwab, and E. Zampelli (1989), "Local Characteristics and Public Production: The Case of Education," *Journal of Urban Economics*, 25, pp. 250--263.

9. Fuss, M. A. and D. McFadden (eds) (1978), *Production Economics: A Dual Approach to Theory and Applications*, Amsterdam: North-Holland.

10. Guikey, D., C. A. Knox Lovell, and R. G. Sickles (1983), "A Comparison of the Performance of Three Flexible Functional Forms," *International Economic Review*, 24, pp. 561--616.

11. Gyimah-Brempong, K. (1986), "Empirical Models of Criminal Behavior: How Significant a Factor is Race?" *Review of Black Political Economy*, 15, Summer pp. 27--43.

12. _____________ (1989), "Production of Public Safety: Are Socioeconomic Characteristics of Local Communities Important Factors?," *Journal of Applied Econometrics*, 4, pp. 57--71.

13. Hamilton, W. B. (1983), "The Flypaper Effect and Other Anomalies," *Journal of Public Economics*, 22 (3), pp. 347--361.

14. Hanushek, E. A. (1986), "The Economics of Schooling," *Journal of Economic Literature*, 24 (3), September, pp. 1141--1177.

15. ___________ (1981), "Throwing Money at Schools," *Journal of Policy Analysis and Management*, 1 (1), Fall, pp. 19--41.

16. ___________ (1979), "Conceptual and Empirical Issues in the Estimation of Education Production Functions," *Journal of Human Resources*, 14 (3), Summer, pp. 351--388.

17. Hausman, J. A. (1978), "Specification Tests in Econometrics," *Econometrica*, 45, pp. 1251--1271.

18.Jimenez, E. (1986), "The Structure of Educational Costs: Multiproduct Cost Functions for Primary and Secondary Schools in Latin America," *Economics of Education Review*, 5, pp. 25--39.

19. Margo, R. A. (1986), "Educational Achievement in Segregated School Systems: The Effects of Separate-but-Equal," *American Economic Review*, 76, pp. 794--801.

20.McElroy, M. B. (1987), "Additive General Error Model for Production, Cost and Derived Demand or Share Systems," *Journal of Political Economy*, 95, pp. 737--757.

21.Panzar, J. C. and R. D. Willig (1977), "Economies of Scale in Multi-Output Production", *Quarterly Journal of Economics*, 91, pp. 481--493.

22. Oates, W. E. (1981), "On Local Finance and Tiebout Model," *American Economic Review*, 71 (2), pp. 93-98.

23. Summers, A. and B. Wolfe (1977), "Do Schools Make a Difference," *American Economic Review*, 67, pp. 639--652.

24. Zellner, A. (1962), "An Efficient Method for Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias," *Journal of the American Statistical Association*, 57, pp. 585--612.