CAPITAL WEALTH INEQUALITY AND PUBLIC BADS: A MATHEMATICAL ANALYSIS

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Rising wealth and income inequality has become a matter of increasing public and professional concern. Trends toward greater inequality have been documented and discussed by numerous authors [Danziger and Gottschalk, 1993; 1995; Wolfson and Murphy, 1998]. Although the main focus of this literature is on the worsening economic condition of the poor and the declining middle class, a frequent subsidiary focus is on the distribution of capital wealth. The substantial increase in capital wealth inequality over the last three decades in the United States has been carefully documented by Wolff [1992; 1994]. In response to the trend, proposals for wealth taxation and/or taxation of inheritances are being discussed, and in some cases advocated [Inhaber and Carroll, 1991; Wolff, 1995]. To date, these advocacies have had little discernible effect on the real world. Indeed, at the time of writing the U.S. Congress is giving very serious consideration to eliminating estate taxation altogether. Nevertheless, seeds planted in the professional literature may ultimately germinate and turn the real-world political tide.

The principal perceived problem with a high level of capital wealth inequality is an equity problem: high inequality in capital wealth ownership generates a high level of inequality in capital property income, and thereby a higher degree of inequality in total household income, consumption, utility, and opportunities for personal advancement. These economic inequalities, especially to the extent that they are perceived to be permanent and impermeable, tend to generate psychological alienation, social discontent and political instability. The extent to which capital wealth inequality is perceived as an equity problem is governed largely by the individual's perception of the nature of capital wealth and capital income. If capital wealth is regarded principally as a legitimate return to hard work and/or entrepreneurial risk-taking, then capital wealth inequality is perceived as less of an equity problem. If, alternatively, capital property income is regarded principally as an unearned rental income accruing to a stock of household capital wealth mostly determined by parental socioeconomic status, financial inheritance and random chance rather than hard work and/or entrepreneurial risk-taking, then it is perceived as more of an equity problem. Numerous economists have addressed these questions both theoretically and empirically [Yunker, 1998; Bowles and Gintis, 2002].

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Throughout modern intellectual history, socialist critics of capitalism have perceived that aside from inequity, a wide variety of additional social problems, such as crime, racism, sexism, militarism, alienation, environmental degradation, and pronounced inequality in effective political power, have been aggravated by capitalistic institutions and conditions. Although some of this criticism may be exaggerated, some aspects could contain more than a kernel of truth. While the collapse of the Soviet Union in 1991 is widely regarded as heralding the end of any serious, organized movement in the contemporary world toward expanded public ownership of business enterprise as a cure for the perceived ills of capitalism, many of the problems traditionally perceived in capitalism by socialists might in principle be ameliorated by various policies short of public ownership of business enterprise—specifically by policies to decrease the inequality of capital wealth ownership.

In contributions on a profit-oriented form of market socialism, Roemer [1993; 1994] argues that equalization of political power would tend to reduce the level of "public bads" (such as pollution) experienced by society, and thereby raise the overall level of social welfare. Roemer's argument to this effect remains relevant even if market socialism itself (even in the relatively conservative profit-oriented form proposed by Roemer) is rejected as too radical—because the argument is not that capitalism *per se* is responsible for a high level of public bads, but rather that a high level of capital wealth inequality is responsible for this. Capital wealth inequality may be reduced by various measures (wealth taxation, inheritance taxation, and so on) short of the abolition of capitalism and the inauguration of socialism. But the question remains whether capital wealth inequality does indeed tend to promote pollution and other "public bads."

Environmental legislation and cleanup efforts in the advanced industrial nations has inspired the hypothesis of an "environmental Kuznets curve", an inverted-U relationship between environmental pollution and per capita income. The basic idea is simple and intuitively plausible: environmental cleanliness is a superior good and once a national economy has completed the transition from low-income to middle-income status, further economic growth will enable its people to afford a higher level of environmental cleanliness. Empirical evidence in favor of an environmental Kuznets curve has been provided by Grossman and Krueger [1995] and Seldon and Song [1994], among others, at the international level, and by Brooks and Sethi [1997] at the domestic level. Evidence against its generality can be found in Moomaw and Unruh [1997] and Harbaugh et al. [2000]. Since the economics profession, after more than 50 years of extensive study and discussion, has not reached a strong consensus on the "distributional Kuznets curve", it is not surprising that no strong consensus has been reached during the much shorter period of time during which the environmental Kuznets curve has been extant. But if we were to accept, for the sake of argument, both the distributional and environmental Kuznets curves, this would imply that beyond middle income, nations with higher per capita income display both lower income inequality and lower environmental pollution, which suggests a positive association, possibly causative in nature, between income inequality and environmental pollution.

Of course, it is a statistical truism that associations need not be causative in nature. And it is easy enough to devise stories where inequality, in and of itself, tends to improve the quality of the environment. To begin with, it is reasonably plausible that the income elasticity of the preference of the individual household for general environmental quality is greater than one. Similarly, it is reasonably plausible that the distribution of effective political power is roughly proportional to the distribution of income. Given these two assumptions, an increase in inequality, holding constant the average household income, would lead to more effective environmental policy and a cleaner environment because the preferences of the richer households for a clean environment would be more decisive in the determination of social policy.

There is a large environmental economics literature on the relationship between pollution and per capita income, but few studies examine the effect of inequality, in and of itself (that is for a given per capita income level), on the amount of pollution. Since both inequality and pollution are considered by most economists to be "bad" in a general philosophical sense, it is perhaps not surprising that in the few studies concerning the issue, there seems to be a tendency toward rationalizing positive causation between the former and the latter. Boyce and his various collaborators have developed a political power theory according to which highly unequal economic status generates highly unequal effective political power, enabling the wealthy minority to shift the environmental costs of pollution onto the less wealthy majority [Boyce, 1994; Torras and Boyce, 1998; Boyce et al., 1999]. The Boyce theory is more or less *a priori* and lacking in an explicit economic foundation (utility maximization by households and so on). But more systematic formal models that also indicate a positive relationship between inequality and pollution have been developed by Magnani [2000] and McAusland [2001].

The Magnani and McAusland models focus on the potential role of income inequality, as opposed to wealth inequality, in the generation of environmental degradation. In his book *A Future for Socialism* [1994], Roemer develops a formal model with the following result: the higher the household's share of capital wealth (θ), the higher will be its preferred level of a profit-enhancing public bad (B) such as pollution. If this result is correct, and if the distribution of effective political power is proportional to the distribution of capital wealth, then an economy characterized by highly unequal capital wealth will also be characterized by a high level of all public bads including pollution.

This article examines the Roemer model conclusion within the context of an alternative model of arguably greater economic content and realism, from which the result is obtained that the effect of the household's share of capital wealth on its preferred level of a public bad is indeterminate in general. However, if specific mathematical forms are assumed for the aggregate production function and the household utility function, some interesting results may be obtained. It is shown that in a Cobb-Douglas economy (both the aggregate production function and the household utility function are Cobb-Douglas forms), the household's share of capital wealth has no effect at all on its preferred level of the public bad. In a constant elasticity of substitution (CES) economy (both the aggregate production function and the household utility function are CES forms), the effect of the household's share of capital wealth on its preferred level of the public bad is governed by the numerical values of the elasticities of substitution for the aggregate production function and the household utility function.

The remainder of this article is organized as follows: First, Roemer's model and result are presented. Then a generalized economic model of the same problem is presented. It would appear that even though the basic generalized economic model is fairly simple, the analysis of this particular problem using the specified model becomes algebraically unwieldy. Therefore, an explicit function version of the same model is developed, using the standard Cobb-Douglas form for both the firm production function and the household utility function. Within this alternative, it is shown by explicit solution that household capital wealth has no effect on its preferred level of the public bad. Since the Cobb-Douglas function is relatively restrictive, additional analysis is carried out using the CES form. In the CES case, it is not possible to obtain an explicit solution of the model, but numerical solutions are obtained that indicate that the effect of the household's share of capital wealth on its preferred level of the public bad is governed by the relevant elasticities of substitution. Following the formal analysis, a brief concluding comment is provided.

THE ROEMER MODEL

In an endnote to Chapter 7 of *A Future for Socialism* [1994, 152-53], Roemer develops a model in which the comparative statics derivative $dB/d\theta$ is necessarily positive. The household utility function is written as follows:

$$u = y - bB^2 \tag{1}$$

where y is income and B is the level of a public bad such as pollution.

This function is maximized with respect to B subject to the budget constraint:

$$y = w + \theta\Pi \tag{2}$$

where θ is the household's share of aggregate profits, and both household wage w and aggregate profits Π are functions of B as follows:

$$w = w(B) \text{ with } w'(B) > 0,\ w''(B) < 0, \tag{3}$$

$$\Pi = \Pi(B) \text{ with } \Pi'(B) > 0,\ \Pi''(B) < 0. \tag{4}$$

The first-order condition for the maximization of u with respect to B is:

$$w'(B) + \theta\Pi'(B) - 2bB = 0. \tag{5}$$

By implicit differentiation of (5), we have:

$$dB/d\theta = -\Pi'/(w'' + \theta\Pi'' - 2b) \tag{6}$$

which by the assumptions above on the w and Π functions is necessarily positive.

The result is formally correct but is based on a somewhat thin and atypical economic foundation. First, the labor-leisure decision of the household is not incorporated into the analysis: leisure does not affect the household's utility and labor does not affect the household's income. The model therefore overlooks a fundamental economic reality. Second, the explicit mathematical form of the household utility function utilized in Roemer's model is nonstandard: not only is it a polynomial, but it is asymmetric in y and B, being linear positive in the former and quadratic negative in the latter. Third, the model lacks a production function and an explicit general equilibrium formulation. It would be of interest, therefore, to ascertain whether the result generalizes to a larger and more economically standard model.

In fairness to Roemer, it should be emphasized that this model was developed within a two-page digressive endnote and does not constitute an important component of the overall argument of *A Future for Socialism*. The Magnani and McAusland models mentioned earlier are far more comprehensive and elaborate than the Roemer model described in this section, and it might therefore be expected that they would provide a better basis for comparison with the model developed herein. However, aside from the fact that the Magnani and McAusland models are concerned with income inequality *per se*, as opposed to inequality in capital wealth ownership, it would be impossible to do justice to their complex expositions within a short space.

Moreover, I believe it is fair to say that despite the considerable technical sophistication displayed in their elaboration, they share some fairly nonstandard fundamental assumptions made by Roemer. For example, Magnani utilizes a linear household utility function in private consumption c and a pure public good Q, representing environmental quality, implying that diminishing marginal utility applies to neither good, and that the elasticity of substitution between the two is infinite. In the McAusland model, effectively no distinction is drawn between production and consumption, in that there is no production side of the model by which the primary factor supplies of the households are translated into the output levels of the "clean" and "dirty" commodities. An interesting aspect of both the Magnani and McAusland models, parenthetically, is that they do not incorporate any assumptions about the higher income segment of the population having greater political power. It is assumed, rather, that policies are determined by the preferences of the median voter. For a constant voter mean income, the higher the level of income inequality, the lower the income of the median voter. If the income elasticity of preference for a clean environment is greater than one, then a higher level of inequality will lead to lower environmental cleanliness by perfectly democratic means.

A GENERAL FUNCTION MODEL

Consider an economy in which there are only two commodities: consumption good Q and labor L, with respective prices p and w. By Walras' Law, one arbitrarily selected commodity may be taken as the numeraire. Take this commodity to be Q, and set $p = 1$. The consumption good is produced by a single firm through a general neoclassical production function in which aggregate capital K and aggregate labor L are standard economic factors of production, and in which the public bad B (for example, pollution) may also be considered an input into the production process:

$$Q = F(K, L, B). \tag{7}$$

The amount of the public bad allowed to the firm is set by government fiat, so that B is a parameter to the firm. We assume a short-run framework in which the amount of capital K is fixed. The amount of aggregate labor L is selected by the firm to maximize gross profit Π, defined as the rental return on capital (rK where r is the interest rate) plus the net operating return on capital $[F(K, L, B) - wL - rK]$:

$$\Pi = Q - wL = F(K, L, B) - wL. \tag{8}$$

Rental return on capital might be defined as interest payments on bonds and other debt instruments, while the net operating return on capital is dividends plus capital gains. For purposes of this analysis, the actual breakdown of gross profit into rental return and net operating return is irrelevant. From the first-order condition for gross profit maximization:

$$\partial\Pi/\partial L = F_L - w = 0 \tag{9}$$

we have the firm's demand for aggregate labor:

$$L^d = L^d(w, K, B). \tag{10}$$

This is a partial equilibrium reduced-form equation. Substituting this function into the production function and then the profit function gives the partial equilibrium reduced-form equation for P:

$$\Pi = \Pi(w, K, B) \tag{11}$$

Turning now to the household sector, each household has a general neoclassical utility function in income y, leisure h, and net public good G:

$$u = u(y, h, G), \tag{12}$$

where

$$y = wl + \theta\Pi, \tag{13}$$

$$h = 1 - l, \text{ and} \tag{14}$$

$$G = 1 - B. \tag{15}$$

Equation (13) is the household budget constraint, equation (14) is the household time constraint, and equation (15) expresses the good G in terms of the bad B. Maximizing utility (12) with respect to household labor l subject to (13), (14) and (15), we obtain the household's supply of labor:

$$l = l(w, \theta, \Pi, B). \tag{16}$$

The households are identical except for the proportion of total financial capital owned by each household (θ). This means that each household confronts the same w, Π and B. Therefore if we use l^i to represent both the labor variable and the labor function for a specific household i, the only argument in the function which needs to be similarly superscripted is θ. Aggregate supply of labor is obtained by summing over all individual households:

$$L^s = \Sigma l^i (w, \theta^i, \Pi, B) = L^s(w, \theta^1, \theta^2,..., \theta^n, \Pi, B) \tag{17}$$

The general equilibrium solution for w is obtained from the equation of the firm's aggregate demand for labor (10) to the household sector's aggregate supply of labor (17). Using the partial equilibrium reduced form equation for Π (11) in the aggregate labor supply equation (17), we have the market equilibrium condition:

$$L^d (w, K, B) = L^s(w, \theta^1, \theta^2,..., \theta^n, \Pi, B) \tag{18}$$

Solving (18) for w, we have the general equilibrium reduced–form equation for w:

$$w = w(K, B, \theta^1, \theta^2,..., \theta^n) \tag{19}$$

By substituting forward from this we obtain the general equilibrium reduced form equation for gross profit Π:

$$\Pi = \Pi(K, B, \theta^1, \theta^2,..., \theta^n) \tag{20}$$

Now consider the problem of household i's preferred level of the public bad B. The general equilibrium reduced-form equations for w and Π have to be substituted directly into the household budget constraint (13), and also indirectly through the effect of these two variables on household labor supply (16). In addition, these two functions go indirectly into the time constraint (14) through their effect on household labor supply.

Making the appropriate substitutions, the utility of household i is expressed as a function of B and the household capital wealth ownership parameters $\theta^1, \theta^2,..., \theta^n$. Using subscript notation for the first partial derivatives of the utility function but full notation for the other partial derivatives, the first-order condition for the maximization of u^i with respect to B is as follows:

$$\partial u^i/\partial B = u^i_y(\partial y^i/\partial B) + u^i_h(\partial h^i/\partial B) + u^i_G(\partial G/\partial B) \tag{21}$$

where

$$(22) \partial y^i/\partial B = [\partial(wl^i + \theta^i\Pi)]/\partial B = (\partial w/\partial B)l^i + w[(\partial l^i/\partial w)(\partial w/\partial B) + (\partial l^i/\partial \Pi)(\partial \Pi/\partial B)] + \theta^i(\partial \Pi/\partial B),$$

$$(23) \partial h^i/\partial B = (\partial h^i/\partial l^i)(\partial l^i/\partial B) = [\partial(1-l^i)]/\partial l^i\,](\partial l^i/\partial B) = -[(\partial l^i/\partial w)(\partial w/\partial B) + (\partial l^i/\partial \Pi)(\partial \Pi/\partial B)],$$

and

$$(24) \qquad \partial G/\partial B = \partial(1 - B)/\partial B = -1.$$

To obtain the effect of the household capital ownership parameter θ^i on the preferred level of the public bad B, we apply implicit differentiation to the first-order condition:

$$(25) \qquad \partial B/\partial \theta^i = -(\partial^2 u^i/\partial B \partial \theta^i)/(\partial^2 u^i/\partial B^2)$$

By the second-order condition for maximization, the second derivative in the denominator of the RHS expression is negative, from which we have the standard comparative statics result:

$$(26) \qquad \text{sign}(\partial B/\partial \theta^i) = \text{sign}(\partial^2 u^i/\partial B \partial \theta^i).$$

Unfortunately, the expression for $\partial^2 u^i/\partial B \partial \theta^i$, if written out fully, is quite complicated. Omitting all terms involving cross second partial derivatives of the utility function, we have, by differentiation of the first-order condition (21) with respect to θ^i:

$$(27) (\partial^2 u^i/\partial B \partial \theta^i) = u^i_{yy}(\partial y^i/\partial \theta^i)(\partial y^i/\partial B) + u^i_y(\partial^2 y^i/\partial B \partial \theta^i) + u^i_{hh}(\partial h^i/\partial \theta^i)(\partial h^i/\partial B) + u^i_h(\partial^2 h^i/\partial B \partial \theta^i)$$

Not counting the first and second partial derivatives of the utility function, this expression contains six partials, two of which ($\partial y^i/\partial B$ and $\partial h^i/\partial B$) are given above by equations (22) and (23) respectively, the first of which contains four component terms and the second of which contains two component terms. With respect to the remaining four partials, the partial $\partial y^i/\partial \theta^i$ contains four component terms, the partial $\partial^2 h^i/\partial B \partial \theta^i$ contains ten component terms, the partial $\partial h^i/\partial \theta^i$ contains two component terms, and the partial $\partial^2 h^i/\partial B \partial \theta^i$ contains four component terms. If expanded fully, therefore, equation (27) would contain 34 terms. And recall that this is without taking into account the cross partial derivatives of the utility function. All of these terms are composed of partial derivatives of general equilibrium reduced-form relationships, the signs of most of which are indeterminate.

Normally, a large number of terms in a comparative statics derivative suggests that the sign of the derivative will be indeterminate because there will likely be conflicts in sign among the terms. It is shown below in the analysis of the CES economy that in the general model the comparative statics derivative is indeed indeterminate since numerical solutions of the CES version of the model show $dB/d\theta^i$ positive for some parameter values and negative for others. Under the circumstances, it is a more practical approach to develop an explicit function version of the model through the same analytical procedure shown above, but using explicit mathematical forms for the firm production function and the household utility function.

A COBB-DOUGLAS EXPLICIT FUNCTION MODEL

Consider an economy in which n households in the economy supply labor to a single profit-maximizing firm which produces Q through a Cobb-Douglas production function in aggregate capital K, aggregate labor L, and a public bad B (for example, pollution). Each household has a Cobb-Douglas utility function in income y, leisure h, and net public good $G = 1 - B$. The households are identical except for the proportion of total financial capital owned by each household (θ).

The specified model is a small-scale general equilibrium model in which the equilibrium w may now be solved for as an explicit function of the parameters. By means of forward substitution, an explicit function may then be obtained relating the utility of each household to the level of public bad B and the household's ownership parameter θ. From this function it is observed that the household's ownership parameter has no effect on the household's utility-maximizing level of B.

The firm's production function relating K, L and B to Q is a Cobb-Douglas form:

(28) $$Q = K^{\gamma}L^{\delta}B^{\xi}.$$

With this production function, the profit-maximizing aggregate labor demand L^d is:

(29) $$L^d = \delta^{1/(1-\delta)}K^{\gamma/(1-\delta)}B^{\xi/(1-\delta)}w^{1/(1-\delta)}$$

Substituting this amount of L into the production function and the profit function, we have the respective partial equilibrium reduced form equations:

(30) $$Q = \delta^{\delta/(1-\delta)}K^{\gamma/(1-\delta)}B^{\xi/(1-\delta)}w^{-\delta/(1-\delta)} \text{ and}$$

(31) $$\Pi = [\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)}]K^{\gamma/(1-\delta)}B^{\xi/(1-\delta)}w^{-\delta/(1-\delta)}.$$

Another quantity which is useful below is Π/w, defined by:

(32) $$\Pi/w = [\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)}]K^{\gamma/(1-\delta)}B^{\xi/(1-\delta)}w^{-1/(1-\delta)}.$$

Each household maximizes (with respect to labor l) a Cobb-Douglas utility function in income y, leisure h and net public good $1 - B$:

(33) $$u = y^{\alpha}h^{\beta}(1-B)^{\mu}$$

subject to the constraints (13) and (14). This gives the household labor supply function:

(34) $$l = \alpha/(\alpha + \beta) - [\beta/(\alpha + \beta)](\theta\Pi/w).$$

Recall that households are identical except for financial capital ownership θ. This means that they have the same preference parameters α, β, μ, and they confront the same economic parameters (determined at the economy-wide level) w and Π. If we differentiate households with an i superscript, only θ and l need be superscripted.

Summing over all n households, the aggregate supply of labor L^s is:

(35) $$L^s = n\alpha/(\alpha + \beta) - [\beta/(\alpha + \beta)](\Pi/w).$$

Equating aggregate demand for labor (23) to aggregate supply of labor (29) and using Π/w from (26) in the latter, we may solve for equilibrium w as:

(36) $$w = [(\alpha\delta^{1/(1-\delta)} + \beta\delta^{\delta/(1-\delta)})/n\alpha]^{1-\delta}K^{\gamma}B^{\xi}.$$

By means of substitution of this into the partial equilibrium reduced form equations for Π/w (32) and l (34), we may derive the following general equilibrium reduced form equations for household income y and leisure h:

(37) $$y = (r + s\theta)K^{\gamma}B^{\xi}$$

where

(38) $$r = [\alpha/(\alpha + \beta)][(\alpha\delta^{1/(1-\delta)} + \beta\delta^{\delta/(1-\delta)})^{1-\delta}/(n\alpha)^{1-\delta}],$$

(39) $$s = [\alpha/(\alpha + \beta)]\{[(n\alpha^{\delta})(\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)})^{1-\delta}]/[\alpha\delta^{1/(1-\delta)} + \beta\delta^{\delta/(1-\delta)}]^{\delta}\} \text{ and}$$

(40) $$h = a + b\theta,$$

where

(41) $$a = \alpha/(\alpha + \beta), \text{ and}$$

(42) $$b = [\alpha/(\alpha + \beta)]\{[(n\alpha)(\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)})]/[\alpha\delta^{1/(1-\delta)} + \beta\delta^{\delta/(1-\delta)}]\}.$$

Note that h does not depend on B. Substituting y and h into the utility function (33), we obtain u as an explicit function of B:

(43) $$u = (r + s\theta)^{\alpha}K^{\alpha\gamma}B^{\alpha\xi}h^{\beta}(1 - B)^{\mu}.$$

Solving the first-order condition for the maximization of u with respect to B for B, we have:

(44) $$B = \alpha\xi/(\alpha\xi + \mu).$$

The household's capital income distribution parameter (θ) is seen to have no effect on the household's optimal level of the public bad B, indicating that $dB/d\theta = 0$. It may be worth pointing out that the finding that a certain comparative statics derivative is zero is a positive, nontrivial result. The "no result" finding is that the comparative statics derivative in question has an indeterminate sign: it may be positive or negative, meaning that it could be zero—but this would be an unlikely special case.

A CES EXPLICIT FUNCTION MODEL

Now consider an economy identical to that studied in the previous section, except that the firm's production function and the households' utility functions are now CES forms as follows:

(45) $$Q = K^{\gamma}(\delta L^{-\rho} + \xi B^{-\rho})^{-1/\rho}$$

(46) $$u = (\delta y^{-\varphi} + \beta h^{-\varphi})^{-1/\varphi}(1-B)^{\mu}$$

where $\sigma_f = 1/(1 + \rho)$ is the firm's elasticity of substitution between L and B and $\sigma_h = 1/(1 + \varphi)$ is the households' elasticity of substitution between income y and leisure h.

In this case the typical household's supply of labor is:

(47) $$l = [1 - (\beta/\alpha)^{\sigma_h} w^{-\sigma_h}\theta\Pi]/[1 + (\beta/\alpha)^{\sigma_h} w^{1-\sigma_h}].$$

The aggregate supply of labor would look the same as the above except that summation over all households eliminates the θ term (since $\Sigma\theta_i = 1$) and replaces 1 with n in the numerator (the number of households).

In the case of the CES production function, it is not possible to solve explicitly for firm demand for labor L^d as a function of wage w. However, the firm's demand for labor is implicit in its first-order condition for profit-maximization with respect to labor, which may be written as follows with w on the LHS:

(48) $$w = K^{\gamma}(\delta L^{-\rho} + \xi B^{-\rho})^{-(1/\rho)-1}\delta L^{-\rho-1}.$$

Similarly, the profits function may be written as a function of L:

(49) $$\Pi = K^{\gamma}(\delta L^{-\rho} + \xi B^{-\rho})^{-1/\rho} - wL.$$

If we substitute equations (48) and (49) into the aggregate labor supply equivalent of equation (47), we would have one equation in one variable L. Unfortunately, this equation is not explicitly solvable for L. However, this particular substitution is indeed convenient for purposes of obtaining numerical solutions of the model by an iterative process. A small computer program was written to find these solutions. In addition, the program uses a simple numerical search routine to determine the B level which maximizes the utility of a given household. For the sake of convenience, the numerical version of the model consists of only two households. Experimentation with the program indicated the critical importance of the elasticity of substitution parameters: the elasticity of substitution within the firm between aggregate labor and the public bad (σ_f), and the elasticity of substitution within the household between income and leisure (σ_h).

The baseline parameter values for the numerical experiments are as follows: $K = 1$, $\gamma = 0.2$, $\delta = .75$, $\xi = .25$, $\alpha = .25$, $\beta = .75$, $\mu = .25$, $\theta^1 = .80$ (the "rich" household), $\theta^2 = .20$ (the "poor" household). A typical numerical result over a range of variation in the two elasticities of substitution in the model, with additional variation in the capital wealth ownership parameters, is shown in Table 1. For each combination of values of the θ parameters, each cell of the table shows results on $B_1 + B_2$ and B_1/B_2 (respectively the total amount of the public bad preferred by the two households combined, and the ratio of the utility-maximizing level of the public bad for the rich household to that of the poor household) for a certain combination of σ_f and σ_h. If the ratio B_1/B_2 is greater than one, this is a case which supports the Roemer hypothesis that capital-wealthy households prefer higher levels of the public bad than do less capital-wealthy households.

The middle cell of each section of the table verifies the analytical result from the Cobb-Douglas version of the model: when the two elasticities of substitution are both unity, the household's capital wealth level has no effect at all on its preferred level of the public bad. The firm's elasticity of substitution between L and B has a qualitative impact on the relative preferences of rich and poor households for the public bad. When σ_f is > 1, the rich household prefers a higher level of the public bad than the poor household. When σ_f is equal to or very near 1, there is very little effect of household capital wealth on preferred public bad level. When σ_f is < 1, the rich household prefers a lower level of the public bad than does the poor household. The situation is more straightforward with respect to the household's elasticity of substitution between income and leisure (σ_h): as this parameter increases, both the rich household and the poor household prefer a lower level of the public bad.

Table 1 also indicates the sensitivity of the result to the existing degree of inequality in capital wealth ownership. The less the inequality, the less difference there is between the preferences of the rich household and the preferences of the poor household for the public bad, regardless of the elasticities of substitution. Indeed, if there is perfect equality in capital wealth ($\theta^1 + \theta^2 = .5$), then the third section of Table 1 shows that the preferred level of the public bad is the same for both households. An interesting aspect of Table 1 is the indication that the sum—as opposed to the ratio—of the amounts of the public bad preferred by the rich household and the poor household is independent of the distribution of capital wealth.

A substantial amount of sensitivity experimentation with the other parameters indicated the robustness of the qualitative result shown in Table 1 for the two elasticity of substitution parameters. In the case of some parameter value combinations, no solution exists to the model, but in all cases in which a solution is obtainable, it is qualitatively as shown in Table 1. The indication is therefore that Roemer's conclusion concerning the positive effect of the household's share of capital wealth on its preferred level of the public bad is true only if the elasticity of substitution between aggregate labor and the public bad is high (greater than 1). An empirical estimation

TABLE 1
Utility-Maximizing Levels of Public Bad for High Wealth Household 1 and Low Wealth Household 2 for Various Elasticities of Substitution and Inequality Levels

Case: $\theta^1 = .9$, $\theta^2 = .1$			
	$\sigma_f = 0.75$	$\sigma_f = 1.00$	$\sigma_f = 1.25$
$\sigma_h = 0.75$	$B_1+B_2 = 0.561$	$B_1+B_2 = 0.500$	$B_1+B_2 = 0.444$
	$B_1/B_2 = 0.753$	$B_1/B_2 = 0.953$	$B_1/B_2 = 1.209$
$\sigma_h = 1.00$	$B_1+B_2 = 0.448$	$B_1+B_2 = 0.402$	$B_1+B_2 = 0.360$
	$B_1/B_2 = 0.792$	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.278$
$\sigma_h = 1.25$	$B_1+B_2 = 0.347$	$B_1+B_2 = 0.313$	$B_1+B_2 = 0.283$
	$B_1/B_2 = 0.846$	$B_1/B_2 = 1.073$	$B_1/B_2 = 1.378$
Case: $\theta^1 = .7$, $\theta^2 = .3$			
	$\sigma_f = 0.75$	$\sigma_f = 1.00$	$\sigma_f = 1.25$
$\sigma_h = 0.75$	$B_1+B_2 = 0.560$	$B_1+B_2 = 0.500$	$B_1+B_2 = 0.447$
	$B_1/B_2 = 0.867$	$B_1/B_2 = 0.976$	$B_1/B_2 = 1.099$
$\sigma_h = 1.00$	$B_1+B_2 = 0.446$	$B_1+B_2 = 0.402$	$B_1+B_2 = 0.362$
	$B_1/B_2 = 0.890$	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.129$
$\sigma_h = 1.25$	$B_1+B_2 = 0.346$	$B_1+B_2 = 0.314$	$B_1+B_2 = 0.285$
	$B_1/B_2 = 0.922$	$B_1/B_2 = 1.039$	$B_1/B = 1.176$
Case: $\theta^1 = .5$, $\theta^2 = .5$			
	$\sigma_f = 0.75$	$\sigma_f = 1.00$	$\sigma_f = 1.25$
$\sigma_h = 0.75$	$B_1+B_2 = 0.560$	$B_1+B_2 = 0.500$	$B_1+B_2 = 0.448$
	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.000$
$\sigma_h = 1.00$	$B_1+B_2 = 0.446$	$B_1+B_2 = 0.402$	$B_1+B_2 = 0.364$
	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.000$
$\sigma_h = 1.25$	$B_1+B_2 = 0.346$	$B_1+B_2 = 0.314$	$B_1+B_2 = 0.286$
	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.000$	$B_1/B_2 = 1.000$

of this elasticity, if it could be done, would assist our judgment on the likelihood that a high level of capital wealth inequality tends to increase public bads such as pollution.

CONCLUSION

The theoretical results obtained here do not prove, of course, that the conclusion derived from Roemer's model (that wealthy capitalists tend to prefer more public bads such as pollution than does the rest of the population) is necessarily empirically incorrect, nor that the conclusion derived from the Cobb-Douglas special case (that wealthy capitalists are no different from the rest of the population in their preferences regarding such things as pollution) is necessarily empirically correct. Indeed, a certain amount of empirical evidence exists to support the proposition that economic inequality is detrimental to environmental quality. For example, Kahn and Matsusaka [1997] find that richer districts in the state of California tend to vote against environmental initiatives more so than poorer districts. Nevertheless, the general form specification of the model suggests that the effect of household capital wealth on the household's preferred levels of public bads is a complicated question, and the numerical results obtained from the CES special case tend to confirm this: numerical analysis of the CES model indicates that the effect can go either way depending on various parameter values. In general, these results suggest that we would need considerably more theoretical and empirical information on the question before we would be in a position to make a relatively strong judgment on it.

The effect of capital wealth inequality on public bads such as environmental pollution is still further complicated by public choice questions. Even if it could be established with certainty that wealthy capitalists have a higher preference for public bads than the rest of the population, this does not necessarily mean that a high level of capital wealth inequality leads to a higher level of public bads. The question remains of how much a relatively small minority of wealthy capitalists is able to influence public choices within a democratic polity. Of course, it is plausible enough that the distribution of effective political power, even within a nominally highly democratic polity, is at least roughly proportional to the distribution of wealth and/or income, and that the preferences of wealthy capitalists would therefore be weighted more heavily in public decision-making than the preferences of low- and middle-income people. For example, it is a well-known empirical fact that the wealthy contribute disproportionately to the campaigns of those running for elective public offices. In addition, the wealthy probably have disproportionate control over the large corporations that dominate the contemporary economy. Corporate behavior regarding polluting externalities is a major area of study in environmental economics, [Kohn, 1998; 1999; Harford, 1987; 1997].

But even if the rich do possess disproportionate political influence, it is a complicated issue whether their influence is sufficiently disproportionate to have a material effect on public policy. The fact that contemporary political democracies are not unduly egalitarian in their policies has long been regarded as a paradox requiring explanation, and not all of the explanations that have been offered depend on a high level of inequality in effective political power. For example, in recent contributions, Roemer [1998; 1999] has put forward the theory that real-world economic and non-economic policies are determined simultaneously in a multidimensional bargaining process, and that equilibrium economic policies are considerably different from what they would be if their determination were not complicated by non-economic issues.

As for the specific issue of present concern, the effect of capital wealth inequality on the level of public bads such as pollution, the complexity of the theoretical literature suggests strongly that this is an empirical issue rather than a theoretical issue, in the sense that *a priori* theoretical analysis is unlikely to yield a strong result one way or the other. The present contribution reinforces this impression. This does not necessarily mean, however, that theoretical analysis does not have an important role to play. Theoretical analysis of these problems provides guidance for empirical investigations by showing what parameters might be important in the determination of the direction of a certain relationship. The present contribution has shown that John Roemer's hypothesis that wealthy capitalists have a higher preference for the public bad than the rest of the population is more likely to be true to the extent that there is a high elasticity of substitution between aggregate labor and the public bad in the production sector of the economy.

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