Eastern Economic Journal, Vol. VI, No. 1, January 1980

On Reading Walras: An Object Lesson for Modern Economists

ROY J. ROTHEIM*

I. Money and General Equilibrium Analysis

It is now incontrovertible that money is inessential in a General Equilibrium economy. The person most responsible for pursuing this point in recent times from within the "scientific community" has been Frank Hahn (1965), (1971), (1973a, b). In his earliest article on the subject, the basis of which was Patinkin's system (1964), Hahn emphasized that "we must show not only that a solution (for the existence of an equilibrium) exists but also that it is one in which money has a positive exchange value." (1965, p. 127) Although couched in mathematical existence proofs, Hahn's criticism is of an economic rather than a mathematical nature. He notes that the specific response of the individual in a general equilibrium framework is that one never suffers from money illusion. In other words, an excess demand for money cannot exist when the price of money is less than or equal to zero. "We are told that the demand for fiat money depends on its exchange value (absence of 'money illusion'). It follows that no money will be demanded if its exchange value is zero." (*ibid.* p. 128) This unique property of fiat money distinguishes it from all other reproducible commodities since existence proofs require that these latter quantities must have positive excess demands even when their respective prices are equal to zero (non-satiety). However, existence proofs require that this property holds for all goods *including money*. "We therefore reach the rather displeasing conclusion . . . that the Patinkin model *always* contains a 'non-monetary' solution. Moreover it is not at once clear how we could establish that it also contains a solution with P_0^* (price of money) >0. . . . Something has gone wrong." (*ibid.*, pp. 128–129)

In one of his later articles Hahn states:

> "It follows that a minimum requirement of a representation of a monetary economy is that there should be transactions at varying dates. An economy which has transactions at every date I shall call a *sequence economy*.
>
> "A good of given physical characteristics available at a given place at a given date may be the subject of transactions at all dates not later than its date of availability. Let us say that transaction dates are *inessential* if the set of equilibria attainable by an economy is not altered by concentrating all transactions at the first date. Accordingly in such an economy money is *inessential* in the sense that no monetary variable need enter into the description, or determination, of that economy's equilibrium." (1973a, p. 230)

Economics as a science obviously must

*Department of Economics, Skidmore College. An earlier version was delivered before a joint session of the American Economic Association and the History of Economics Society Fall 1978, and subsequently at seminars at Lehigh and Temple Universities. My single debt is to Professor William Jaffé, not only for saving me from some embarrassing slips, but more significantly, for setting a model for scholarship that all historians of economics emulate and revere. It is to him that this article is dedicated.

resolve this anomaly. One way of finding our way out is to remind ourselves of how we got in. Such is the purpose of this article. I shall attempt to present Léon Walras's argument from the *Elements of Pure Economics* (1926) that attempts to justify the role of money in a general equilibrium framework. The purpose of this exercise is two-fold: First, Walras's model, while crude in many ways as judged by modern standards, is still, in my opinion, one of the most explicit and intellectually honest presentations of the subject. At each juncture of his presentation, Walras, after making his goal explicit, makes sure that the reader is aware of the assumptions (zero time of production, no uncertainty, trading in *bons* before actual trades become effective, etc.) necessary to obtain those goals. Thus, by working through his model, we can have a better idea of where to seek the ways out of the above anomaly. Second, once completed, I wish to briefly show that the routes chosen by modern general equilibrium theorists may lead to dead ends and that we may have to look elsewhere, such as in the Keynesian and Post-Keynesian models, to solve the riddle of a monetary economy.

II. Time and Money in Walras's Analysis

Walras raised two distinct points in the lessons on "The Mechanism and Equations of Circulation and Money." The first was that production took time, and the second was that all exchanges were assumed to transpire by the medium of money. The equilibrium for the entire system is assumed from the outset. Production, offer, payment, and delivery must now all begin.

The circulation of the equilibrium quantities runs into a serious problem in a time dimensional context. In order for an individual to provide his labor services for the week, he will have to eat. However, the nourishment that he requires for the duration of this week will take, say, three months to produce, process, and transport to his local grocery store. If this situation holds for all individuals, then no one will be able to provide his labor services until he was eaten, but no one will be able to eat unless he provides labor services to produce society's groceries. The same dilemma holds, as well, in the case of raw materials, circulating and fixed capital.

How does Walras propose to include money in his system? The problem just stated above is one of synchronization of payments and receipts. If the equilibrium quantities (the flow of products and capital goods proper produced in the *current* period) are determined prior to their circulation, it is simply necessary to determine the equilibrium quantities of consumers' goods and services, fixed and circulating goods proper, and raw materials, to be held over from *last* period in order to avoid the synchronization problem. However, what is the necessity of money in such an environment? As will be seen, Walras never gave a suitable answer to this question. He merely stated that all payments, evaluated in terms of *numeraire,* were effected through the medium of money. No other explanation was provided.

Given this paltry *raison d'etre* for the existence of money, Walras postulated that consumers must have a fund of circulating capital consisting of final products on hand so that they can feed themselves prior to the receipt of wages, rent, and interest payments; cash to purchase final products from other consumers; and savings held in the form of money. Producers require finished products to put on display, raw materials to begin production as soon as the equilibrium is determined, and finally a quantity of cash to replace *his* stocks "and for the purchase of productive services while waiting to be paid for the products he has sold." (*ibid.*, p. 318) Circulating capital and money are held by all capitalists

and are loaned either to themselves in their capacity as consumers or to entrepreneurs to be used in the production process. Circulating capital and money are also demanded by consumers and entrepreneurs for their services of availability.[1] They provide a service to consumers and entrepreneurs because they add continuity to the process of consumption and production. For this service, demanders of circulating capital and money are willing to pay a premium.[2] Quoting Walras:

> "(C)irculating capital and money (will) make it possible for productive services to be transformed into products instantaneously, provided that the consumers pay the interest charges on the capital required for this sort of transformation." (*ibid.*, p. 242)

Walras defined capital "as the sum of total fixed and circulating capital goods hired, not in kind, but in money, by means of credit." (*ibid.*, p. 317. But, see also p. 270 §235) Money is constantly being made available in the form of savings through two separate means: by entrepreneurs repaying the money equivalent of circulating capital to capitalists, and by capitalists purchasing capital goods which may be theoretically represented as so many perpetual annuity shares." If Walras could posit a theory to explain the demand for money, then he could derive "the theory of the determination of prices in terms of money." (*ibid.*, p. 257)

The above, however, carries the analysis way ahead of itself. First, the equilibrium analysis for the services of availability of final products, raw materials, and circulating and fixed capital, must be completed *in kind* before it may be postulated that these stocks are held in the form of money. If it can be shown that this equilibrium is entirely self-contained, then the question may be asked: What is the necessity of money in such a framework? In the following, the analysis will intentionally deviate from Walras's presentation so that it may be shown that "tacking" money onto a system of real exchange does not establish the existence of a monetary economy.[3]

Quantities of final products, raw materials and capital goods proper that perform the functions of circulating capital, i.e., that provide their bearers with services of availability, are distinguished by primes, i.e., they appear as $A', B', C', D', \ldots, M', T', P', K' \ldots$ "Letting (a) or (A') be the numeraire ... so that $1, p_b, p_c, p_d, \ldots p_m \ldots p_t, p_p, p_k \ldots$ are again the prices of commodities of all sorts in terms of (A) we shall now let $p_{a'} = i$,[4] $p_{b'} = p_b i \ldots, p_{m'} = p_m i \ldots$ be the prices of the services of availability of $(A'), (B') \ldots (M') \ldots$, just as $\Pi_t = p_t i$, $\Pi_p = p_p i$, $\Pi_k = p_k i \ldots$ are the prices of the services of (T), (P), $(K) \ldots$" (*ibid.*, p. 319)

Utility functions, $r = \phi_{a'}(q), r = \phi_{b'}(q) \ldots$, exist for the services of availability of circulating capital (A'), (B'), ... The equation of exchange or budget constraint appears as:

$$o_t p_t + o_p p_p + o_k p_k + \cdots + o_{a'} p_{a'} + o_{b'} p_{b'} + \cdots + q_m p_{m'}$$

[1]According to Patinkin (1964), Walras's term *services d'approvisment* has been erroneously interpreted by Jaffé as 'services of availability'; Instead, Patinkin believes that it should be read literally as 'services of storage.' Since the purpose of circulating capital was to facilitate circulation, Jaffé's interpretation seems to better conform with the spirit of Walras's intent.

[2]These "interest charges" should be considered as charges payable not only to outside lenders of capital, but also to owners of capital lending it to themselves. I am grateful to William Jaffé for this point.

[3]For a somewhat similar interpretation, written independently during the same week, the reader is directed to Jaffé (1978 pp. 27–32)

[4]If we allow commodity A to also be money, which Walras defined as U, then the equation $P_{a'} = i$ implies that in equilibrium, the cost of holding money for its services of availability should equal that foregone perpetual income that could be received if this amount were invested in a capital good.

$$+ \cdots = d_a + d_b p_b + d_c p_c + d_d p_d + \cdots + d_e p_e.^{5} \quad (1)$$

Utility functions are maximized subject to the equation of exchange (1) to yield the following "equations of maximum satisfaction":[6]

$$\begin{aligned} \phi_{a'}(q_{a'} - o_{a'}) &= p_{a'}\phi_a(d_a) \\ \phi_{b'}(a_{b'} - o_{b'}) &= p_{b'}\phi_a(d_a) \\ &\cdots\cdots\cdots \end{aligned} \quad (2)$$

Individual offer functions can be derived from (2) represented as:

$$\begin{aligned} o_{a'} &= f_{a'}(p_t, p_p, p_k, \cdots \\ &\qquad p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_{m'}, \cdots p_e) \\ o_{b'} &= f_{b'}(p_t, p_p, p_k, \cdots \\ &\qquad p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_{m'}, \cdots p_e) \end{aligned} \quad (3)$$

Aggregating over all individuals yields "the following (m) equations of total effective offer": (*ibid.*, p. 320)

$$\begin{aligned} O_{a'} &= F_{a'}(p_t, p_p, p_k, \cdots \\ &\qquad p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_{m'}, \cdots p_e) \\ O_{b'} &= F_{b'}(p_t, p_p, p_k, \cdots \\ &\qquad p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_{m'}, \cdots p_e) \end{aligned} \quad (4)$$

[5]The symbols t, p, k, and m signify the flow of services of the stocks of capital T, P and K and raw materials M. Commodities are represented by the symbols a, b, c, and d. The symbol e is used by Walras to denote the quantity of a fictitious commodity, E, that individuals purchase when they choose to purchase fewer commodities than are equal to the value of their income. It is said that they purchase shares of perpetual net income.

It should be noted that many of the equations in this paper do not exactly conform with those appearing in the *Elements*. This follows since we have intentionally purged Walras's equations of the money element.

[6]Equation (2) simply states that utility is maximized when the ratio of marginal utilities between any commodity or commodity's service of availability and the numeraire (A) equals the ratio of their prices (recall that $P_A = 1$).

At this point in the analysis, Walras derived the effective offer of money. However, its enumeration will be postponed until the equilibrium of circulating capital traded *in kind* is completed. On the demand side, Walras postulated the following coefficients of production for the services of availability of circulating capital: $a_{a'}, a_{b'} \ldots a_m \ldots b_{a'}, b_{b'}, \ldots b_m \ldots ma', m_{b'}, \ldots, m_m \ldots k_{a'}, k_{b'}, \ldots k_m$ ... "required for the production of (A), (B) ... (M) ... (K) ..." (*ibid.*, p. 322) The following m + s equations relating "equality between the demand and offer of the services (A'), (B') ..." (*ibid.*) and M may be stated as:

$$\begin{aligned} &a_{a'}(D_a + D_{a'}) + b_{a'}(D_b + D_{b'}) \\ &\quad + \cdots + M_{a'}(D_m) + \cdots + k_{a'}D_k \\ &\quad + \cdots = O_{a'} \\ &a_{b'}(D_a + D_{a'}) + b_{b'}(D_b + D_{b'}) \\ &\quad + \cdots + M_{b'}(D_m) + \cdots + k_{b'}D_k \\ &\quad + \cdots = O_{b'} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &a_m(D_a + D_{a'}) + b_m(D_b + D_{b'}) \\ &\quad + \cdots + M_m(D_m) + \cdots + k_m D_k \\ &\quad + \cdots = O_m \end{aligned} \quad (5)$$

Walras also assumed that the price of each commodity equalled its cost of production:

$$\begin{aligned} &a_t p_t + a_p p_p + a_k p_k \\ &\quad + \cdots + a_{a'}p_{a'} + a_{b'}p_{b'} \\ &\quad + \cdots + a_m p_{m'} + \cdots = 1 \\ &b_t p_t + b_p p_p + b_k p_k \\ &\quad + \cdots + b_{a'}p_{a'} + b_{b'}p_{b'} \\ &\quad + \cdots + b_m p_{m'} + \cdots = p_b) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &m_t p_t + m_p p_p + m_k p_k \\ &\quad + \cdots + m_{a'}p_{a'} + m_{b'}p_{b'} \\ &\quad + \cdots + m_m p_{m'} + \cdots = p_m \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

$$\begin{aligned} &k_t p_t + k_p p_p + k_k p_k \\ &\quad + \cdots + k_{a'}p_{a'} + k_{b'}p_{b'} \\ &\quad + \cdots + k_m p_{m'} + \cdots = p_k \\ &\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned} \quad (6)$$

The aggregate excess of income over consumption, E, i.e., the supply of savings (demand for new capital goods) equals:

$$\begin{aligned} &D_k P_k + \cdots + D_{a'} + D_{b'}p_{b'} \\ &\quad + \cdots + D_m p_m + \cdots = F_e(p_t, p_p, p_k, \cdots \\ &\qquad p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_m, \ldots, i) \end{aligned}$$

Since the rate of net income[7] must also be equal for all circulating capital, the following $m + s$ equations result:

$$1 = \frac{P_{a'}}{i}, p_b = \frac{P_{b'}}{i} \ldots, p_m = \frac{P_{m'}}{i} \quad (7)$$

Solving the system, there are $3m + 2s$ equations consisting of:

(a) m equations of total effective offer of circulating capital,

(b) $m + s$ equations relating equality between the demand and offer of circulating capital, and

(c) $m + s$ equations indicating the equality of the rate of net income for all circulating capital, to solve for the $3m + 2s$ unknown quantities;

(a) "the m ... quantities exchanges of the services of circulating capital goods (A'), (B') ..." ...

(b) "the $m + s$ ... prices of the services of circulating capital goods (A'), (B') ..., (and) raw materials (M) ..." ...

(c) "the $m + s$ quantities manufactured of circulating capital goods and raw materials." (*ibid.*, p. 323)

If these results are added to Walras's systems of exchange, production, and capital and credit, a complete system of general equilibrium will exist *without any notion of money*. If such is the case, then what role does money play in a system such as Walras's?

According to Walras, individuals do not demand physical quantities, but instead, they demand money with which physical goods and services may then be purchased[8] (see Jaffe [1978 pp. 21–22]). Therefore, individuals possess utility functions: $r = \phi_\alpha(q)$, $r = \phi_\beta(q)$, ..., $r = \phi_\epsilon(q)$ ... for the services of availability of final products (A'), (B') ... and perpetual net income (E') "not *in kind*, but *in money*." (*ibid.*, p. 320) Money, denoted by the symbol, U, has "a price of its own, p_u, and a price for its service of availability $p_{u'} = p_u i$." (*ibid.*)

With the inclusion of money into the system, the equation of exchange (1) is now amended to read:

$$\begin{aligned} &o_t p_t + o_p p_p + o_k p_k \\ &\quad + \cdots + o_{a'}p_{a'} + o_{b'}p_{b'} + \cdots + q_m p_{m'} \\ &\quad + \cdots + o_u p_{u'} = d_a + d_b p_b \\ &\quad + d_c p_c + d_d p_d + \cdots + d_e p_e. \end{aligned} \quad (1')$$

Maximization of utility functions with respect to the equation of exchange (1′) yields the following equilibrium equations:

$$\begin{aligned} \phi_\alpha(\alpha) &= p_{a'}\phi_a(d_a) \\ \phi_\beta(\beta) &= p_{b'}\phi_a(d_a) \\ &\cdots\cdots\cdots \end{aligned}$$

[7]Net income, Π, is defined, according to Walras, by the equation:

$$\Pi_{kj} = p_{kj} - (u_{kj} + v_{kj})P_{kj}$$

where p_{kj} is the gross income derived from the services of any particular capital good, j, while u_{kj} and v_{kj} are charges for depreciation and loss resulting from accidents, that is, a form of insurance premium where u and v are calculated as some percentage of the market price of the capital good proper, P_{kj}. From this, Walras defined the rate of net income, i, where:

$$i = \frac{p_{kj} - (u_{kj} + v_{kj})P_{kj}}{P_{kj}} \qquad (j = 1, \ldots, L)$$

which in equilibrium will be equivalent for all capital goods.

[8]If relative prices have not been cried, then how do the individuals know how much money to demand?

$$\phi_\epsilon(\epsilon) = p_{a'}\phi_a(d_a) \qquad (2')$$

"from which we obtain, first, the quantities desired of services (A'), (B') ... (E') (in the form of money)

$$\alpha = f_\alpha(p_t, p_p, p_k, \cdots p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_m, \cdots p_{u'}, p_e).$$

$$\beta = f_\beta(p_t, p_p, p_k, \cdots p_b, p_d, p_d, \cdots p_{a'}, p_{b'}, \cdots p_m, \cdots p_{u'}, p_e).$$

. . . . . .

$$\epsilon = f_\epsilon(p_t, p_p, p_k, \cdots p_b, p_c, p_d, \cdots p_{a'}, p_{b'}, \cdots p_m, \cdots p_{u'}, p_e). \qquad (3')$$

secondly, the value of these quantities expressed in terms of numeraire:

$$\alpha p_{a'} + \beta p_{b'} + \cdots + \epsilon p_{a'}$$

and finally the quantity of (the services of) money effectively offered:

$$o_u = q_u - \frac{\alpha p_{a'} + \beta p_{b'} + \cdots + \epsilon p_{a'}}{p_{u'}} \qquad (4')$$

(*ibid.*, p. 321)

Summing over all individuals yields the aggregate effective offer of the services of availability of money:

$$O_u = Q_u - \frac{d_\alpha p_{a'} - d_\beta p_{b'} + \cdots + d_\epsilon p_{a'}}{p_{u'}} \qquad (4'')$$

The second term on the right hand side of equation (4") constitutes what Walras has called "desired cash-balances" (*ibid.*) The term O_u, on the left hand side of the equation, is that quantity of money that individuals desire not to retain from their original endowments of money.

The demand for the services of availability of money is derived from equation (5). Just as with aggregate net offers (or demand) for circulating capital and money, entrepreneurs demand the services of availability (A'), (B') ... (M) ... (K) described by the system of equations:

$$\alpha_{a'}(D_a + D_{a'}) + \beta_{a'}(D_b + D_{b'}) + \cdots + \mu_{a'}D_m + \cdots + \chi_{a'}D_k \cdots = \delta_\alpha$$

$$\alpha_{b'}(D_a + D_{a'}) + \beta_{b'}(D_b + D_{b'}) + \cdots + \mu_{b'}D_m + \cdots + \chi_{b'}D_k \cdots = \delta_\beta$$

. . . . . .

$$\alpha_m(D_a + D_{a'}) + \beta_m(D_b + D_{b'}) + \cdots + \mu_m D_m + \cdots + \chi_m D \cdots = \delta_\mu$$

. . . . . .

$$\alpha_k(D_a + D_{a'}) + \beta_k(D_b + D_{b'}) + \cdots + \mu_k D_m + \cdots + \chi_k D_k \cdots = \delta_\chi \qquad (5')$$

where "$\alpha_{a'}, \alpha_{b'} \ldots \alpha_m \ldots \alpha_k \ldots \beta_{a'}, \beta_{b'} \ldots \beta_m \ldots \beta_k \ldots \mu_{a'}, \mu_{b'}, \ldots \mu_m \ldots \mu_k \ldots \chi_{a'}, \chi_{b'} \ldots \chi_m \ldots \chi_k$ (are) the coefficients of production made up of the services (A'), (B') ... (M) ... (K) ... required *in money* and not *in kind* for the production of (A), (B), ... (M) ... (K) respectively" (*ibid.*, p. 322); and $\delta_\alpha, \delta_\beta, \ldots \delta_\mu \ldots \delta_\chi \ldots$ are the aggregate demands for the services of availability of (A'), (B') ... (M) ... and (K) ... in the form of money.

By a particular method of composition, Walras defined the "total amount demanded of the services of money for productive purposes expressed in terms of numeraire." (*ibid.*):

$$a_u(D_a + D_{a'}) + b_u(D_b + D_{b'}) + \cdots + m_u D_m + \cdots + k_u D_m + \cdots = \delta_\alpha p_{a'} + \delta_\beta p_{b'} + \cdots + \delta_\mu p_{m'} + \cdots + \delta_\chi p_k + \cdots \qquad (6')$$

Equating (4") with (6') results in the "equality between the demand and offer of the services of money (U)." (*ibid.*, p. 323)

$$\frac{\delta_\alpha p_{a'} + \delta_\beta p_{b'} + \cdots + \delta_\mu p_{m'} + \cdots + \delta_\chi p_k + \cdots}{p_{u'}} = O_u$$

$$Q_u - \frac{d_\alpha p_{a'} + d_\beta p_{b'} + \cdots + d_\epsilon p_{a'}}{p_{u'}} \qquad (7')$$

Equation (7') may be rewritten as:

$$Q_u = \frac{D_\alpha}{p_{u'}} + \frac{\Delta_\alpha}{p_{u'}} + \frac{E_\alpha}{p_{u'}} \qquad (8')$$

where $D_\alpha/p_{u'} = (d_\alpha p_{a'} + d_\beta p_{b'} + \ldots)/p_{u'}$ equals the demand for the services of money by consumers; $(\Delta_\alpha/p_{u'})$ equals the demand for the services of money by producers; and $(E_\alpha/p_{u'})$ equals the demand for money savings. This can be further reduced to:

$$Q_u = \frac{H_\alpha}{p_{u'}} \qquad (9')$$

where $H_\alpha = D_\alpha + \Delta_\alpha + E_\alpha$.

Equation (9') describes what is customarily referred to as the quantity equation or the quantity theory of money. H_α in that equation refers to society's desired cash balances, while the term, Q_u, represents the actual quantity (stock) of money in existence.

Once the equilibrium price for the services of money has been established and, along with it, the equilibrium rate of perpetual net income, the price of money:

$$P_u = \frac{P_{u'}}{i} \qquad (10')$$

can be established immediately.

In order to understand why Walras believed equation (9') to represent the quantity theory of money, the following question must be raised: What will be the effect on the system if there is an increase in the quantity of fiat money, Q_u? According to Walras:

> "If perchance $Q_u P_{u'} = H_\alpha$ the question would be completely settled. Generally, however, we find that $Q_u P_{u'} \gtrless H_\alpha$ and the problem is to determine how equality between the demand and offer of money is reached by groping through adjustments in $P_{u'}$. . . . If we first suppose (general) economic equilibrium to be established, then the equation of monetary circulation would be solved almost without any groping, simply by raising or lowering $P_{u'}$ according as $Q_u \gtrless H_\alpha / P_{u'}$ at a price $P_{u'}$ which has been cried at random. . . . Thus: The price of the service of money is established through its rise or fall according as the desired cash balance is greater or less than the quantity of money." (*ibid.*, pp. 326–27)

The whole analysis is contingent upon the invariance of the term H_α with respect to changes in the price of the services of money, $P_{u'}$. That is, it must be shown that after all adjustments have taken place, the real and monetary sectors are independent. To see the potential relationship between the two variables, it will be necessary to retrace the derivation of the individual demands for the services of availability of money, $\alpha, \beta, \ldots, \epsilon$ and $d_\alpha, d_\beta, \ldots, d_\epsilon$ which make up the term D_α in the composite term, H_α. Therefore, variations in $P_{u'}$ directly affect the level of the demand for the services of money, H_α.

Walras averts this anomaly by postulating that the demand curve for the services of availability of money is a rectangular hyper-

bola. Thus, $o_u p_{u'}$ will everywhere be the same for any variation in $p_{u'}$.

"If we assume this to be the case . , , , then . . . $o_u p_{u'}$ will not vary at all, and the previously established equilibrium will remain unchanged in consequence to a change in $p_{u'}$ alone." (*ibid.*, p. 328)

In all fairness, Walras does note that this is a special case and that in the general case there might be some interdependence between changes in $p_{u'}$ and H_α:

"We must admit, however, that the dependence of these items (d_α, d_β, . . . , d_ϵ) on $p_{u'}$ is very indirect and very weak. . . . We may, therefore, enunciate with what amounts to *almost* rigorous exactness that: The rarete or value of the service of money is . . . inversely proportional to its quantity." (*ibid.*, pp. 327–329 italics added)

Such is the development of the *almost* quantity theory of money. In response to the hard facts of the real world, Walras responds that:

"The foregoing conclusions which embody the so-called quantity theory, i.e. the law of inverse proportionality between the value and the quantity of money, were arrived at by the deductive method. . . . Economists who challenge the quantity theory generally base their arguments on observation and history, thus arriving at their conclusions by the inductive method. They are, however, compelled to recognize that observation and history show striking instances of fluctuations in the value of money that are the inverse of fluctuations in its quantity. . . . For example, the issue of 30 to 40 millards in assignats in France from 1789 to 1796 lowered the value of the medium of exchange in the proportion of 100 to 2.5 or 3. This grandiose experiment cannot be repeated as often as would be necessary to convince the opponents of the quantity theory; and that is why it is particularly fortunate that economics is a science in which the process of reasoning makes up for the ambiguities and the deficiencies in our experience." (*ibid.*, pp. 366–367)

It has been shown that Walras went to great lengths to incorporate money into his system. Individuals demand cash-balances in order to facilitate the exchange of consumer goods and services, capital goods and services, and savings: *all of which are known with perfect certainty prior to the time of circulation.*[9]

In this model, expectations are always fulfilled due to the existence of trading in tickets prior to the commitment of resources. A brief digression on Walras's *tatonnement* theory may be instructive at this point.

The process of *tatonnement* that Walras envisioned was rather straightforward and he was certainly not the first to have postulated such an adjustment mechanism. However, as in the case of the equilibrium conditions, Walras believed that his was the first mathematically deductive exposition of the problem.[10]

Tatonnement, according to Walras, solves "in practice, in the market by the mechanism of free competition . . . the same problem to which we have given a theoretical solution in exchange" (*ibid.*, pp. 241–242) production, capital formation, circulating capital and money.

In the instance of pure exchange "our task is very simple: we need only show that the upward and downward movements of price solve the system of equations of offer and demand by a process of groping (*par tatonnement*). (*ibid.*, p. 170) The price of any commodity (relative to the numeraire) which was in excess demand (supply) would rise (fall) until the stated equilibrium was attained. In production, by an adjustment mechanism analogous to that in pure exchange, both prices of final consumers' goods and services as well as those of productive services would rise or fall relative to the numeraire until equilibrium was reached. Adjustment in capital formation required all of the above plus variations in the flow of new capital goods by variations in their prices along with variations in the rate of perpetual net income so that the demand and supply of shares of perpetual net income would be equated.

Finally, in the case of circulating capital and money, it is required that the price of the services of money adjust until the demand for the services of money is equated with the supply of the stock of fiat money in existence.

Walras, however, perceived an anomaly in the system. This anomaly related to the fact that in the real world production was irreversible. Walras noted that:

"(T)he process of groping (*tatonnement*) in production entails a complication which was not present in the case of exchange. In exchange, (the total existing quantities of) commodities do not undergo any change. When a price is cried, and the effective demand and offer corresponding to this price are not equal, another price is cried for which there is another corresponding effective demand and offer. In production, productive services are transformed into products. After certain prices for services have been cried and certain quantities of products have been manufactured, if these prices and quantities are not the equilibrium prices and quantities, it will be necessary not only to cry new prices *but also to manufacture revised quantities of products.*" (*ibid.*, p. 242 italics added)

What is observed here is that production, unlike exchange, requires the commitment of resources. If perchance, a non-equilibrium price vector is cried, then exchange and production at this false price vector implies not only false trades but also the production of false quantities. Once a revised price vector is cried, re-exchange as well as the production of new final consumers' goods and services are required. This latter requirement implies that all goods produced at the originally cried price vector must now, somehow, be decomposed without cost into their constituent means of production in order to facilitate the production of new quantities at the newly cried vector of prices.

Walras's perception of the anomaly was that such a situation although leading eventually to an equilibrium would certainly not result in the equilibrium that was postulated in the mathematical system of equations and unknowns. It was, however, of such magnitude that it was necessary for him to quash its existence if he was to maintain his particular thesis. He, therefore, chose the following fiction rather than analyzing the bothersome complexities of the real world:

"In order to work out as rigorous description of the process of groping (towards equilibrium) in production as we did in exchange . . . we have only to imagine, on the one hand, that entrepreneurs use tickets (*bons*) to represent the successive quantities of products which are first determined at random and then increased or decreased according as there is an excess of selling price over cost of production or vice versa, until selling price and cost are equal; and on the other hand, that landowners, workers, and capitalists also use tickets to represent the successive quantities of services (which they offer) at prices first cried at random and then raised or lowered according as there is an excess of demand over offer or vice versa, until the two become equal." (*ibid.*)

Deferring the actual commitment of resources until the equilibrium price vector has been cried and all demand and supply prereconciled expunges the general equilibrium analysis of the distinction between notional and realized prices and quantities. With the advent of tickets, it is seen that all activity transpires at the point of equilibrium. This, in turn, rules out the notion of time in the historical sense of the term. Logical time exists in that it "takes time" for the market participants to trade in tickets. However, yesterday, today, and tomorrow are all quoted only in terms of equilibrium values and they are thus historically empty. Once the time of production is minimized, Walras merely

[9]According to Jaffé's Collation Note [a] to part VI of Walras (1926, p. 601): "The first important change [in monetary theory] occurred in ed. 2 when Walras substituted the notion of an 'encaisse desiree' [desired cash-balances] for his earlier notion of a 'circulation a desservir' [mechanism for circulation]."

[10]On the whole matter of Walras's theory of *tatonnement* see Jaffé (1967).

assumes that "productive services (can be) transformed into products *instantaneously* provided that the consumers pay the interest charges on the capital required for this sort of transformation." *(ibid.)*

Returning to the main argument, along with the disavowal of false trades, is the culling out of the notion of uncertainty. "If," said Walras, "we remove this possibility of change for a given period of time, and if we suppose the prices of goods and services and also the dates of their purchase and sale to be known for the entire period, we eliminate all occasion for uncertainty." (*ibid.*, p. 318)

In such a model, can there be any need for money? According to Professor Hicks, "It . . . seems . . . that people, if they could only foresee the future development of prices, would have no need for holding money." (1933, p. 447) Patinkin, however, in disagreement with Hicks on this point, states that "This . . . criticism of Walras . . . is not at all a vital one. For as Marget has pointed out, 'even in a world in which everything were perfectly foreseen, a lack of synchronization between the receipt of income and its outlay would give rise to a need for cash balances so long as there are not perfect facilities for the borrowing of money in anticipation of receipts and the investment of money during the period elapsing between receipt and outlay.' " (1964, pp. 548–549)

Patinkin's view and by implication, Marget's, (1935) is in error. In the Walrasian equilibrium system, the entire future is collapsed into the present. The flow of capital goods and their requisite services are determined for the entire future by means of adjustment in the rate of perpetual net income which equated net savings out of capitalized wealth by means of individual purchases of perpetual net income (and thereby capital goods) with the flow of produced capital goods.

By intentionally dichotimizing Walras's chapter on circulation and money, it has been shown that individuals and capitalists will hold previously produced consumers' goods and services as well as capital goods and services as long as they are willing to pay the interest charges on the (circulating) capital to tide them over until their receipts on the sale of their labor power and final goods come through. The equilibrium quantity of circulating capital to be held in stock prior to the production and exchange of the flow of new commodities is sufficiently determined *without the intervention of money as a medium of circulation (exchange).*

III. After Walras

It was illustrated in the last section that Walras held a *belief* in the neutrality of money with respect to the real sector equilibrium. This is a question that relates to the stability of the equilibrium. More important, however, is the question of existence. It was shown above that we are hard pressed in our attempt to justify the existence of money in Walras's system. If our interpretation of Walras's reasoning for the inclusion of money into the economy is correct, that is to facilitate the *circulation* of commodities and savings, then by consciously dichotomizing his system such that an equilibrium for goods and circulating capital could be posited, *in kind*, it follows that money transactions are redundant and provide no additional information.

The reasons for this redundancy of money, as was shown above, are threefold: (1) all transactions for the entire future are made in the present; (2) these transactions are the equilibrium transactions; and (3) there is no "quantity" uncertainty in the future.

In this final section I shall briefly consider first the attempts by modern orthodox theorists to explain the existence of money in an equilibrium framework and then to offer some comments on an alternative route for describing the role of money in an economy.

We begin again with Hahn (1973a, b). Assume two economies, one such as Walras's, where all transactions for the entire future are collapsed into the present described by a unique price vector p^*; the other a *sequence* economy where markets exist at every date, each having a price vector p^t. "If in this enlarged economy equilibrium is attained when q^t is proportional to p^* as viewed from t, then nothing will have altered by allowing transactions at every date." (Hahn, 1973a, p. 230)

Most research has recently centered around sequence economies. In a sequence economy, it can be posited that there must be some means to store wealth from spot market to spot market. Also, in such an economy, costs to activate a market may exist, i.e. there are set-up and other transaction costs. It should be apparent that these are the same issues raised by Walras in his attempt to justify the existence of money. With regard to the former, Hool states that it can

> "be shown that the short run equilibrium price of money will be positive provided that traders have a desire to transfer wealth from the present to the future and that money is both a store of value and the institutional medium of exchange." (1976, p. 440)

Of the latter consider the following proposition of Heller and Starr:

> "Transactions costs often possess the set-up property or some other form of diminishing marginal cost (i.e., a nonconvexity). . . . Such scale economies in the execution of transactions provide one of the main motives for holding inventories of households and firms. In a sequence economy where money acts as a medium of exchange, non-convex transactions costs of individuals will provide motivation for holding idle balances of money, i.e. inventories of the medium of exchange. The willingness to hold idle balances is essential to obtain an equilibrium in an economy with a non-zero money supply and hence such behavior is a cornerstone of monetary theory." (1976, p. 195)

Recall the statement attributed to Hahn made toward the beginning of the paper concerning the *inessential* role of money in a general equilibrium economy. If this argument is elaborated, it can be seen that an equivalent sequence economy can only be established if three conditions hold (Hahn 1973a, b): (1) that no *sequential learning* in the sense of Radner (1972) exists;[11] (2) that there is no price uncertainty at all dates; and (3) that all markets that exist today will exist in the future (complete markets). If these three conditions hold, money is then *unessential* in this sequence economy: "The money of this construction is only a contingent store of value and has no other role. *Moreover, its existence is fortuitous since there is nothing which demands the sequential structure which will necessitate the introduction of such a store.*" (Hahn 1973a, p. 231 italics added) Thus, the *raison d'etre* of money in sequence economies such as those proposed by Hool, and Heller and Starr is invalid. Just as their reasons for including money into the economy parallel Walras's, so too does the criticism of their choice hold true. Progressing from a static economy to a sequence economy is a step in the right direction, but not if the latter is isomorphic with the former. Then the same three criticisms (see xxxxx above) of the general equilibrium model remain valid and money remains a redundant element. Hahn sums it up by stating:

> "After all this I can state with some precision what it is that I have in mind when I claim, as I now want to claim, that the foundations of monetary theory have not yet been laid. The position of

[11] Hahn defines the absence of sequence learning as a case where "the sequence of markets and the extra prices should not make possible more information on the environment than was available when transactions were concentrated in the first period." (1973a, p. 231) See also Hahn (1973c, section IV).

formal theory on this matter can be summed up as follows: the representations of the monetary economy used, are either isomorphic to an inessential economy, or if not (as in the case of Patinkin) give no account of either the role of money or the sequential character of their construction. But the inessential economy does not need money and one must give reasons for grafting on to it monetary constraints." (1973a, p. 233)

Is there an alternate route? A possible answer may be found in the writings of recent post-Keynesians. The purpose of much of the research in post-Keynesian theory has been to explain three factors associated with the performance of the monetary function. These are: (1) the presence of uncertainty (as opposed to risk); (2) the consequences of disappointment in light of an uncertain future; and (3) the conditions under which market clearing institutions will or will not exist.

It is immediately obvious that these three factors stem directly from dissatisfaction with the general equilibrium model both in its static and in its sequence form. These three conditions are the direct opposites of the conditions necessary for a sequence economy equilibrium: "When these stringent restrictions are dropped it will not be clear . . . how to characterize the equilibrium of an economy." (Hahn, 1973a, p. 236) This, as will be seen below, may indeed be the essence of the whole issue; the barrier that holds theorists back from the attainment of a meaningful monetary theory. This barrier may be the reliance on the notion of *equilibrium* as a necessary goal to theoretical pursuit.

The idea of a sequential economy is critical to post-Keynesian theory. The sequence, however, takes place in historical rather than logical time. In a world of historical time, as Mrs. Robinson states:

"Time, so to say, runs at right angles to the page at each point on the curve. To move from one point to another we would have either to rewrite past history or to embark upon a long future. In dynamic conditions, changes in the composition of demand, changes in technique, and changes in costs of specific factors of production are continuously going on. Investments are always made in less than perfect knowledge of present possibilities and less than perfect confidence in expectations about the future. The stock of capital in existence today is not that which would have been chosen if the future, that is now today, had been correctly foreseen in the past. It is not composed of units of the most appropriate technique; it contains numerous fossils from earlier periods of techniques which were chosen in conditions different from those obtaining today." (1971, p. 104)

If each time an individual makes an actual commitment of resources which alters his views concerning his decision making process, then the axiom of rationality gives no clues as to the next move that the individual will make (even though it is known that whatever decision he does make will be rational at the next moment in time). Only in a world where preferences are timeless, unchanging, and known with perfect or, at least, actuarial certainty, will the rationality postulate provide a basis for understanding the decision making process. (See Davidson 1972, Rotheim 1979, and Shackle 1955).

In a world of uncertainty, individual economic agents, although not necessarily recognizing the potential assymetry of time and action, make decisions to commit resources which will, in effect, carry them to an *unknown* position(s) in the future. Under such circumstances, each disequilibrating action will cause the decision-maker to reevaluate his past, current, and future status which, in turn, causes him to alter the basis of his decisions. The fact that the new decision will also be rational is irrelevant. Once the newly revised decision is effected, the action itself will create a new set of circumstances which again will cause the individual to reappraise and redefine his decision set on future actions. No equilibrium state is attainable since the activities of individuals are now seen as a process in which each action leads to a new set of circumstances which, in turn, carries individuals forward to new and uncertain positions in the future. The fixed point needed for existence proofs in the equilibrium model disappears as the action occurs because the preferences that result were not a point on the original n-dimensional space of the transformation. There is no relationship between the point of activity and the point of equilibrium since the latter changes as the former is effected while the resultant distance between the two is non-quantifiable.

Stated in the terminology of the above argument, *uncertainty* implies that a sequential economy is *essential* and that agents are subject to sequential learning at each moment in time. When such an occurrence is the rule, the economy cannot be operated under the assumptions of the pure *tatonnement* system. Rather, false trades and false production become the rule by which the economy operates. This is a critical point from which post-Keynesian theory proceeds.

Clower (1965) observed that in a world of uncertainty the asset base upon which agents make future commitments may change. Some agents may be happily surprised to find that there have been windfall gains attached to their implemented decisions. Others, however, might be disappointed by receiving a benefit which is significantly less than what was originally anticipated. What will be the effects of these windfall gains and losses on future decisions? Furthermore, will agents be able to shake off their losses without any serious repercussions so that they may continue to make their *normal* decisions in the future? To quote Jaffé:

"In the mathematical solution [of the tatonnement process], the parameters to be held constant are not only *tastes* (utility functions or preference scales), not only the total *resources* (total quantities of each of the different commodities in the theory of pure exchange, or total quantities of available productive services in the theory of production), not only the *technology* (relevant only to the theory of production), but also the *distribution of wealth among the traders* (that is, 'the values of the sum of the (initial) quantities possessed' by each of the traders, actual or potential). When this last parameter is allowed to shift, as it must when trading at 'false prices' takes place, the equilibrium (if it exists) is unlikely to remain unchanged." (1967, p. 4)

The research being done by Shackle (1952), (1955), (1972), and Davidson (1972) with regard to what Shackle calls "crucial" experiments is most significant. Cruciality implies that the commitment of resources which amount to a significant proportion of an agent's total assets (both current and future) may, if disappointed, result in the inability of the agent to reperform the experiment in the near future, if at all. When these situations occur, individuals attempting to ameliorate their mistakes may affect each other's activities so as to exacerbate the mitigating circumstances which caused the original miscalculation, thus making the situation worse for both the individual as well as the collection of all persons.

Dropping the perfect certainty and non-cruciality assumptions implies that, in a decentralized market economy, crucial mistakes can have disastrous consequences on individual agents as well as on the economy as a whole. Therefore, it is necessary for the economy to establish institutional arrangements so that the deleterious effects of uncertainty and cruciality may be partially minimized. It is in this context that money and monetary institutions become relevant. (Cf. Vickers 1975, 1978.)

The most important function of money is that it is the most liquid of all assets. An individual with a definite expected series of purchases to be made in the near future is aware that a certain quantity of the medium of exchange is required to complete these transactions. In a world of uncertainty, the form in which wealth is stored until that date is now important. That asset in which wealth

is stored must first and foremost possess those properties which enable it to have a well-organized continuous spot market (see Keynes 1936 chapter 17). In other words, it must be highly liquid.

The liquidity requirement immediately rules out assets such as fixed capital goods from being a store of value. If an individual, being paid today, decides to spend this income in equal sums at weekly intervals, then it is irrational to purchase e.g. four machines to be resold on successive Fridays for the medium of exchange. Although an additional flow of income may be obtained from productively employing these capital goods, a highly unlikely occurrence, when it comes time to convert these assets into the medium of exchange, it will be a very rare instance in which it will be possible to obtain the original purchase price. Here the capital loss would far exceed the income from utilization resulting in a negative return for the individual. To quote Davidson:

> "Any durable good—an automobile, a lathe, even an overcoat—has a store of value quality in the sense that it can be carried over to the future. Nevertheless, since the spot markets for most durables (especially fixed capital and consumer durables) are so poorly organized and discontinuous (if they exist at all) because of destandardization, high carrying costs, and the absence of a financial institution to 'make' the spot market that the costs of converting such durables into money at any future date are very high *and* uncertain." (1972, p. 194)

In deference to fixed capital goods, individuals who desire not to consume immediately may wish to store their wealth in the form of titles to capital goods or other placements since these assets are more liquid than capital goods proper owing to their possession of well organized second hand markets. Treasury Bills, Negotiable Certificates of Deposit, and common shares are a few forms of liquid short-term assets. Each of these placements possesses a well-organized second hand market. Faced with a choice between storing wealth in money or in some form of placement, would not a rational individual always choose the latter form? Not in a world of uncertainty. An individual cannot predict, today, what the spot prices of these placements will be when future commitments come due. Thus, for transactions purposes individuals must store a portion of their wealth in perfectly liquid form, i.e. in money, for it is only money which can be held without fear of capital loss. It is therefore seen that in a world of uncertainty, the existence of money as an institution provides individuals with a degree of confidence that they will be able to complete future transactions as they come due. As Keynes once said, "The possession of actual money lulls our disquietude."

How can this view of the role of money be reconciled with the general equilibrium theory presented above? I doubt that such a reconciliation is possible. The reason is fundamental. Money, in the post-Keynesian framework, is a dynamic institution. Its origins are endogenous to the growth of the system. As the economy grows, so too does the importance of money and monetary institutions grow. It is within such a *non-equilibrium* framework that money should be discussed.

References

Clower, R. (1965) "The Keynesian Counter-revolution: A Theoretical Appraisal" in Hahn and Brechling, eds. *The Theory of Interest Rates* (London) pp. 103–125.

Davidson, P. (1972) *Money and the Real World* (London).

Hahn, Frank (1965) "On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy" in F. Hahn and F. Brechling, eds. *The Theory of Interest Rates* (London) pp. 126–135.

——— (1971) "Equilibrium with Transactions Costs" *Econometrica* vol. 39, no. 3.

——— (1973a) "On the Foundations of Monetary Theory" in M. Parkin and A. Nobay, eds, *Essays in Modern Economics* (New York).

——— (1973b) "On Transaction Costs, Inessential Sequence Economies and Money" *Review of Economic Studies*, vol. 40, no. 3.

——— (1973c) *On the Notion of Equilibrium in Economics: An Inaugural Lecture* (CUP).

——— (1973d) "The Winter of Our Discontent" *Economica*, August.

Heller, W. P., and R. Starr (1976) "Equilibrium with Non-Convex Transactions Costs: Monetary and Non-Monetary Economies" *Review of Economic Studies*, vol. 43(2).

Hicks, J. R. (1933) "Gleichgewict und Konjunctur" *Zeitschrift fur Nationalokonomie*, Band 4, Heft 4.

Hool, B. (1976) "Money, Expectations and the Existence of a Temporary Equilibrium" *Review of Economic Studies*, vol. 43(3).

Jaffé, W. (1967) "Walras's Theory of Tatonnement: A Critique of Recent Interpretations" *Journal of Political Economy*, vol. 75.

Jaffé, William (1968) "Walras's Economics As Others See It" (manuscript August 25, 1978)

Keynes, J. M. (1936) *The General Theory of Employment Interest and Money* (London) reprinted in *The Collected Writings of J. M. Keynes* (hereafter *JMK*) vol. VII (Cambridge 1971).

——— (1937) "The General Theory of Employment" *Quarterly Journal of Economics* February 1937; reprinted in *JMK* vol. XIV pp. 109–123.

Marget, A. (1935) "Monetary Aspects of the Walrasian System" *Journal of Political Economy* vol. 43 pp. 145–186.

Marx, Karl (1859) *Contribution to a Critique of Political Economy* (Chicago 1908).

Patinkin, D. (1964) *Money, Interest, and Prices* 2nd ed. (New York).

Radner, R. (1972) "Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence Market" *Econometrica*.

Robinson, J. (1971) *Economic Heresies* (New York).

Rotheim, R. J. (1979) "On the Nature of Production and Contracts in an Arrow-Debreu Economy." *Journal of Post-Keynesian Economics*.

Shackle, G. L. S. (1952) *Time in Economics* (Amsterdam).

——— (1955) *Uncertainty in Economics* (CUP).

——— (1972) *Epistemics and Economics* (CUP).

Vickers, D. (1975) "Finance and False Trading in Non-Tatonnement Markets" *Australian Economic Papers* vol. 14 (December).

——— (1978) *Financial Markets in the Capitalist Process* (Philadelphia).

Walras, L. (1926) *Elements of Pure Economics*, translated and annotated with collation of editions by William Jaffé (Homewood 1954).