Eastern Economic Journal, Vol. VIII, No. 3, July 1982

A Note on Individual Utility Maximization in a Household Context*

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Economists often assume a household can be treated as a single utility maximizing unit. The "new home economics," which deals specifically with household decisions about fertility and labor supply, assumes a "harmonious household" (i.e., household utility maximization is always consistent with utility maximization of individual household members). [Terleckyj 1976] Ironically, some of the same investigators have analyzed marital instability, the existence of which suggests that not all households maintain or perhaps ever achieve such harmony. [Becker, Landis and Michael 1977] Models of households with individual utility-maximizing members in (potential) conflict are a recent innovation. Manser and Brown (1980) and McElroy and Horney (1981) posit households in which conflict can arise between individual utility maximizers, but in these models, household members do not "care" about each other.[1]

The purpose of this paper is to present a simple general model of a household in which members can "care" about each other in the Becker (1974) sense; yet conflict is possible. We show in passing that Becker's conclusions about harmonious households, including the "rotten kid" theorem, do not hold in our general case.

Becker defines the head of household as an individual who "cares" sufficiently about other household members to transfer income or other resources to them.[2] Caring is manifested in the head's utility function:

$$U_i = U(X_i, U_j) \qquad (1)$$

where U_i, X_i = utility and consumption of head i

and U_j = utility of the other household members, $j = 1, \ldots, n$

The utility of the other members appears in the head's utility function, and in a purely consumption setting, we replace U_j with X_j, the consumption of other household members:

$$U_i = U(X_i, X_j) \qquad (2)$$

A key implicit assumption of this formulation is that "caring," as defined by eqs. (1) and (2), is the only possible motive that the head may have for transferring income to other household members. Suppose, however, that we modify the utility function of the head so that he gets some satisfaction from the act of *transferring* income rather than just from the

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This research was funded by a grant from the National Institute of Child Health and Human Development. Helpful comments were received from Bryan Boulier and an anonymous referee.

[1]Manser and Brown introduce "caring" indirectly with an efficiency parameter on own consumption, while McElroy and Horney ignore it completely.

[2]Becker (1974) p. 1074. In order to be the head, a household member must also have sufficient income to make the transfers. Caring is a necessary but not sufficient condition to be the head. An interesting implication of this definition is that if another family member's income rises fast enough the head could be "beheaded."

pleasure the income produces for the recipient.[3] Consider a specific example of this kind of "action" utility: the husband (h) derives some satisfaction from his role as the family "breadwinner." The addition of this form of action utility implies the following changes in eq. (2) for a two member household:

$$U_h = U(X_h, X_w, A) \quad (3)$$

$$A = T/Y_f \qquad O \leq A \leq 1 \quad (4)$$

where T = transfer of income from husband (h) to wife (w) and

$$Y_f = \text{family income}$$

Assume that all of the arguments of this function have a positive but diminishing effect on utility. For the purpose of illustration only, we also assume the wife is selfish in the sense that her utility depends only on her own consumption or

$$U_w = U(X_w) \quad (5)$$

The family budget constraint is

$$Y_f = Y_h + Y_w = p(X_h + X_w) \quad (6)$$

where p is the price of consumption, which is set equal to one.

We begin the analysis by considering the effect of the inclusion of A in the husband's utility function on the distribution of consumption in the household. We can compare the marginal utility functions for the husband with respect to both the husband's and wife's consumption for equations (2) and (3). To simplify the analysis suppose that the husband is the sole "breadwinner" so that $Y_f = Y_h$ and $X_w = T$.

In equation (7) we compare the marginal utility of the husband's consumption (MU_h) from eq. (2) with the marginal utility of husband's consumption (MU_h^*) from eq. (3) with action utility:[4]

$$MU_h \equiv \frac{dU_h}{dX_h} > \frac{dU_h}{dX_h} + \frac{\partial U_h}{\partial A} \cdot \frac{\partial A}{\partial X_h} \equiv MU_h^* \quad (7)$$

Since the interaction between A and X_h is negative (i.e., an increase in the husband's consumption necessarily implies a reduction in the transfer), the husband's marginal utility from own consumption is lower with action utility.

By the same token, the marginal utility of the wife's consumption is higher with action utility:

$$MU_w \equiv \frac{dU_h}{dX_w} < \frac{dU_h}{dX_w} + \frac{\partial U_h}{\partial A} \cdot \frac{\partial A}{\partial X_w} \equiv MU_w^* \quad (8)$$

We know $(\partial A/\partial X_w)$ is positive, and thus $MU_w^* > MU_w$. In order to reach equilibrium, the husband will increase X_w and reduce X_h. This is shown diagrammatically in Figure 1. The vertical axes measure the marginal utility of consumption. The length of the horizontal axis is family income, $Y_f = X_h + X_w$. The husband's consumption and marginal utility is read from left to right and the wife's is read from right to left. With no action utility, MU_h and MU_w are the relevant schedules, which intersect at point E, with husband consuming AC and wife CD. We know from equations (7) and (8) that action utility reduces MU_h^* below MU_h and raises MU_w^* above MU_w. In this case, the new equilibrium is at E*, with the husband consuming AB and the wife BD.

We now consider two further changes in the standard approach: first, a redistribution of family earnings, and second, an increase in the wife's earnings. According to Becker, a simple redistribution of family earnings should have no effect on household welfare. However, if utility function (3) is substituted for (2), then a redistribution of income leads to a reduction of U_h. This result can be proved by considering the husband's alternatives when income is redistributed. If he chooses to maintain T and thus A, X_h must decline (since $X_h = Y_h - T$) and X_w would increase. Since at the original levels of X_h and X_w $MU_h = MU_w$, the effect of reducing X_h and increasing X_w must be to reduce U_h. The husband's alternative is to maintain X_h, but in this case both T and A decline and so U_h must also decline. Thus the existence of action utility implies that changes in the household distribution of income do affect individual welfare.

A second type of disturbance to the family's initial equilibrium occurs when the wife's opportunity wage rises sufficiently to induce her to want to enter the labor force.[5] We first consider the possible impact of such a change on the husband's utility by totally differentiating (3):

$$dU_h = \frac{\partial U_h}{\partial X_h} dX_h + \frac{\partial U_h}{\partial X_w} dX_w + \frac{\partial U_h}{\partial A} \left(\frac{\partial A}{\partial T} dT + \frac{\partial A}{\partial Y_h} dY_h + \frac{\partial A}{\partial Y_w} dY_w \right) \quad (9)$$

and then evaluating the effect of a change in Y_w:

$$\frac{dU_h}{dY_w} = \frac{\partial U_h}{\partial Y_h} \frac{dX_h}{dY_w} + \frac{\partial U_h}{\partial X_h} \frac{dX_w}{dY_w} + \frac{\partial U}{\partial A} \left(\frac{\partial A}{\partial T} \frac{dT}{dY_w} + \frac{\partial A}{\partial Y_w} \right) \quad (10)$$

where dY_h/dY_w is assumed to be zero. All of the partial derivatives are presumed positive except for $\partial A/\partial Y_w$, which is negative. The sign of equation (10) is thus ambiguous because some of the expressions on the right hand side are positive and others are negative. We are interested in showing the possibility that dU_h/dY_w is negative, so the discussion is limited to those cases. If dT/dY_w is zero, dX_w/dY_w is also zero. In this case dU_h/dY_w is negative if

$$\frac{\partial U_h}{\partial A} \frac{\partial A}{\partial Y_w} > \frac{\partial U_h}{\partial X_w} \frac{dX_w}{dY_w}$$

If, on the other hand, dT/dY_w is negative, then dX_h/dY_w must be positive. In this case dU_h/dY_w is negative as long as the absolute value of the first two terms of (10) exceeds the absolute value of the third. In both of these cases the decision by the wife to enter the labor force leads to a reduction in the husband's utility. On the other hand, if the wife is selfish (i.e., see (5)) she does not take into account the loss of U_h. The result of this change is a disagreement, or the "rotten wife" syndrome.[6] Even if the wife does take her

[3] Leibenstein (1976, Ch. 11) presents several related hypotheses about intrahoushold conflict which lead, of course, to X-inefficiency in consumption. Becker himself (1974, sec 3.c) introduces merit goods, in which the utility from T depends on the specific goods in X_w.

[4] $$\frac{dU_h}{dX_h} = \frac{\partial U_h}{\partial X_h} + \frac{\partial U_h}{\partial X_w} \cdot \frac{dX_w}{dX_h}$$

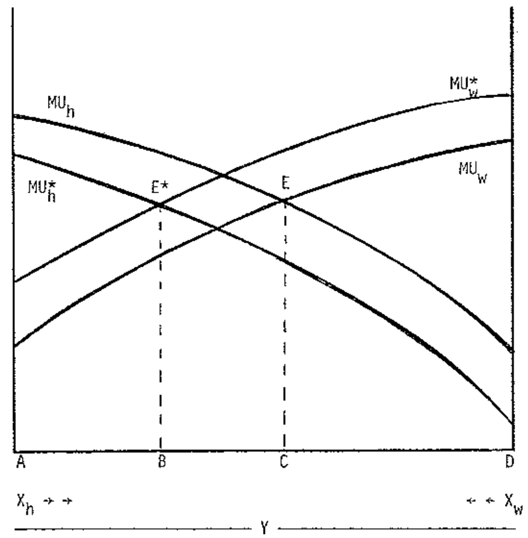

Figure 1

[5] We are implicitly assuming that leisure is one component in the composite consumption good X. The rise in the opportunity wage, then, increases pure consumption by enough to offset the loss in utility from the decrease in leisure.

[6] The wife may take into account her husband's utility loss in devising a strategy for negotiating a resolution to the conflict, but this does not necessarily mean that she actually suffers a loss of utility because her husband does.

husband's loss of utility into account (i.e., $U_w(X_w, X_h)$) the net effect of an increase in the opportunity wage may still be an increase in her utility and thus disagreement.

Two interesting results emerge from this analysis. First, Becker's "rotten kid" theorem does not hold in the presence of action utility. A "rotten kid" or, as in our example, a "rotten wife," is a household member who is selfish in the sense that she derives utility only from her own consumption and does not "care" about the consumption levels of other household members (see eq. (5)). The theorem states that as long as there is a head of household (as defined above), utility maximization by selfish individuals is consistent with family utility maximization. Our analysis shows that this result may not always hold if the act of transferring income or exercising power, by itself, produces utility. A change in the level or distribution of family income may negatively affect a household member by reducing his control over family resources.

Second, we have taken a step toward realism in the modeling of household behavior. Caring is a key part of a household, but disagreements are just as important. Our generalization of the Becker household permits disagreements to arise in the presence of caring. In a specific application of the concept of action utility (Seiver and Cymrot, 1981) we have analyzed household disagreements over desired family size. Many other specific applications and extensions remain on the research agenda, including household decisions about location and labor force participation.

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