

AGGREGATE PRODUCTION FUNCTIONS – A PERVASIVE, BUT UNPERSUASIVE, FAIRYTALE

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In the hit play, *Proof*, there is a description of a rock band made up of mathematicians. They play a number called “ i ” in which they just stand around the stage for some minutes doing nothing. That is because i is an imaginary number. In the same spirit, I am tempted to conclude any discussion of aggregate production functions now. Indeed, it is truly amazing that, after so many years, we should be having a symposium on aggregate production functions; for, perhaps even more than the square root of negative one, aggregate production functions are truly imaginary.

Nevertheless, economists go on behaving as if there were no problem here, and even some of those most firmly opposed to the existence of aggregate production functions (not the participants in this symposium), implicitly base their criticisms of neo-classical economics on the violation of insights whose validity rests on the existence of aggregate production functions. Hence a symposium on this imaginary topic is not only required, but is all too likely to be ignored.

I begin this discussion by briefly reviewing the case against aggregate production functions and then consider its implications, some of which are discussed in the papers of this symposium.¹

For simplicity, I assume that well-behaved production functions exist at the level of the firm. There are then two types of aggregation problems involved in the existence of an aggregate production function for the economy as a whole or even for some sector thereof. The first of these consists of aggregation over different factors to form aggregate labor or aggregate capital and of aggregation over different products to form aggregate output. This set of problems exists even at the level of the firm.

The second problem consists of aggregation over firms. That is trivial if all factors and all outputs are continually re-assigned to firms to maximize efficiency, but this does not happen in practice (except, perhaps, when comparing situations of long-run competitive equilibrium). In particular, different types of capital goods are associated with different techniques, and such capital goods are relatively fixed in place rather than constantly reallocated over firms.

There exists a large and pretty comprehensive literature on these problems. (I suppose that, by now, I am pretty much the grand old man of that literature, but I am far from the only contributor.) Briefly, an examination of the conditions required for aggregation yields results such as:

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- Except under constant returns, aggregate production functions are unlikely to exist at all.
- Even under constant returns, the conditions for aggregation are so very stringent as to make the existence of aggregate production functions in real economies a non-event. This is true not only for the existence of an aggregate capital stock but also for the existence of such constructs as aggregate labor or even aggregate output.
- One cannot escape the force of these results by arguing that aggregate production functions are only approximations. While, over some restricted range of the data, approximations may appear to fit, good approximations to the true underlying technical relations require close approximation to the stringent aggregation conditions, and this is not a sensible thing to suppose.

The papers that make up this symposium add another fundamental point – one that has also been in the literature for some time. When one works – as one must at an aggregate level – with quantities measured in value terms, the appearance of a well-behaved aggregate production function tells one nothing at all about whether there really is one. Such an appearance stems from the accounting identity that relates the value of outputs to the value of inputs – *nothing more*.²

All these facts should be well known. They are not, or, if they are, their implications are simply ignored by macroeconomists who go on treating the aggregate production function as the most fundamental construct of neoclassical macroeconomics. Yet the implications of the points I have listed are not merely theoretical. As this symposium's papers show, they include:

- The specification and estimation of the aggregate demand curve for labor;
- The measurement of productivity and, especially, the interpretation (or, perhaps more properly, the *misinterpretation*) of the Solow residual; and
- The use of aggregate production functions to validate the neoclassical theory of distribution.

To these I would add generally the interpretation of such concepts as “investment”, “capital”, “labor”, or “gross domestic product” in public policy discussions or any other context that supposes them to be related as inputs or outputs in a true production relationship. We are talking here of the very foundations of neoclassical macroeconomics.

Make no mistake, however. We are *not* talking about the foundations of neoclassical *microeconomics*. Nor are we talking about the foundations of the neoclassical theory of distribution. These have nothing whatever to do with the existence of aggregate production functions which would be, if they existed, a convenience, at best.

I mention this because some of the opponents of neoclassical theory explicitly or implicitly suppose that the reverse is true. Moreover, they do not recognize that they are doing so.

In particular, following Piero Sraffa, the Sraffians consider the existence of reswitching and reverse capital deepening to be a decisive criticism of neoclassical theory. They believe that this was the deciding factor in the Cambridge debates over capital theory. But that view fails to realize the following. Reswitching and reverse capital deepening only appear paradoxical if one supposes that aggregates should behave the way intuition suggests they should behave – the way that factors of production and outputs behave at the micro level. But the non-existence of aggregate production functions means that such intuition simply does not apply. No further consequence can be read from its failure.

I conclude with the following remarks. The consequences of the non-existence of aggregate production functions have been too long overlooked. I am reminded of the story that, during World War II, a sign in an airplane manufacturing plant read: “The laws of aerodynamics tell us that the bumblebee cannot fly. But the bumblebee does fly, and, what is more, it makes a little honey each day.” I don’t know about bumblebees, but any honey supposedly made by aggregate production functions may well be bad for one’s health.

Further, I am informed (by Jesus Felipe) that attempts to explain the impossibility of using aggregate production functions in practice are often met with great hostility, even outright anger. To that I say (as I have before in a different area of debunking [Fisher, 1984, 509]), that the moral is: “Don’t interfere with fairytales if you want to live happily ever after.”

ENDNOTES

1. For a detailed survey of the issues, see Felipe and Fisher [2003] and Fisher [1992-93].
2. This is related to an old result of mine that, in simulation experiments, a Cobb-Douglas aggregate production function appears to work whenever factor shares are relatively constant, even though such an aggregate does not represent the underlying technological relationships at all well. Indeed, it explains the puzzle of some of those experiments that, while finding that same phenomenon, no similar organizing principle occurred when experimenting with CES functions. See Fisher [1971] and Fisher, Solow, and Kearl [1977].

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