IN SEARCH OF A FAIR BET IN THE LOTTERY

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INTRODUCTION TO LOTTERY GAMES

"Lotto" is among the most popular games offered by state lottery associations, accounting for roughly one-quarter of total revenues for state-run U.S. lotteries in the late 1990s and early 2000s. As of July 2005, 40 states had lotteries, and every state association offered some version of a lotto game either through their own game or through a multi-state association such as the twenty-seven member Multi-State Lottery Association (Powerball) or the twelve state Mega Millions association.

Lotto games generally consist of an individual picking a set of five or six numbers from a group of approximately 35-55 choices. Winning numbers are then randomly selected at a weekly or bi-weekly drawing. A player whose ticket matches all of the winning numbers wins the jackpot prize while players matching some, but not all, of the winning numbers win smaller consolation prizes. In part, lotto derives its popularity from the large jackpot prizes that can be won in this game. While other lottery games such as instant tickets, numbers, or keno might offer top prizes ranging from \$100 to \$100,000, lotto games typically advertise jackpot prizes starting at \$1 million or higher.

The jackpot prize is funded by allocating a percentage of ticket sales to the jackpot prize pool. If no ticket matches the winning numbers, the money in the pool is carried over into the next drawing and is added to the allocated funds from ticket sales in the next period. Because the jackpot prize pool is allowed to roll-over in this manner, the grand prize can become quite large if no one hits the jackpot in a large number of successive periods. Indeed, advertised jackpots exceeding \$50 million are quite common, and occasionally lotto jackpots have been known to exceed \$250 million.

Because lotto is one of the few games of chance where the expected return varies with each drawing, these games have been widely studied in the academic literature, and the theory on buyer behavior and ticket payoffs is well-established. While numerous researchers have proposed the possibility that under specific conditions the lottery may present bettors with a "fair bet," that is, a gamble with a positive expected return, their conclusions are generally based either solely on supposition or on the examination of just one or two lotto games. This paper uses expected payoff functions

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developed in other research to answer the empirical question of whether lotteries actually are ever fair bets based on an extensive data set. In addition, the notion of whether it is ever possible to earn positive expected returns from purchasing every possible number combination is explored in depth.

EXPECTED PAYOFFS FROM LOTTERIES

Testing whether lotto games present a fair bet requires an estimate of the expected return from the purchase of a lottery ticket. Several researchers have presented estimates of this expected return starting with Clotfelter and Cook [1989] and including DeBoer [1990], Krautmann and Ciecka [1993], Shapira and Venezia [1992], Scott and Gulley [1993, 1995], and Matheson [2001].

Since the price of a lotto ticket and the odds of winning remain fixed regardless of the size of the jackpot, it is natural to assume that the expected return of purchasing a lotto ticket will increase along with the size of the jackpot. The complicating factor, however, is that as the advertised jackpot grows, the number of ticket buyers typically increases as well. The increased number of ticket buyers increases the probability that the winning numbers will be shared by two or more tickets. Thus, the increase in expected return due to the increase in the size of the jackpot is tempered by the prospect of potentially having to share this larger jackpot among several winners. While this phenomenon does reduce the expected value of a ticket, as shown by Matheson and Grote [2004], ticket purchases almost never rise so rapidly in the face of a high jackpot that the expected value of a ticket falls despite an increase in the jackpot. In other words, if the jackpot is not won during a particular drawing, the jackpot in the next period will increase due to additions to the jackpot pool and the expected value of a ticket will increase due to this higher jackpot. The expected value of the ticket will rise more slowly than the jackpot, however, because of the increase in ticket sales that higher jackpots generally generate. Still, the observed net change in the expected return will almost always be positive.

Following Matheson [2001], who presents the most detailed function, the expected return, ER_t, from the purchase of a single lottery ticket with randomly selected numbers is shown in equation (1).

$$(1)\quad ER_t = \left[\sum^{i} w_i V_{it} + (AV_{jt} / dvr_t)(1 - e^{-B_t w_j}) / B_t \right](1-\theta) + \left[\sum^{i} w_i + w_j \right]\theta\tau$$

where w_i is the probability of winning lower-tier prize i, V_{it} is the cash value of lower-tier prize i at time t, w_j is the probability of winning the jackpot prize, AV_{jt} is the advertised jackpot prize at time t, dvr_t is a divisor used to convert the advertised annuitized jackpot into a net present value, B_t is the number of other ticket buyers for the drawing in period t, θ is the tax rate, and τ is the price of a ticket.

It is a fact that certain combinations of numbers (multiples of 7, birthdays, vertical or diagonal columns on the play slip, etc.) are more commonly played than other combinations, and therefore by playing rarer combinations a ticket buyer can earn an expected return above this average expected payout. The ability to earn above normal returns is limited by the amount to which the distribution of numbers played deviates from a uniform distribution. Since roughly 70 percent of all lotto tickets sold use computer generated numbers, which can be reasonably assumed to follow a uniform distribution, any supernormal expected returns are limited to the deviation from uniformity by the 30 percent of tickets that are sold to players who select their own numbers. Furthermore, as lotto jackpots grow, the percentage of players selecting their own numbers falls, further reducing any ability of players to select advantageous numbers during periods of high expected returns. Still, the expected value in equation (1) should be seen as lower bound for the game. See Clotfelter and Cook [1989, p. 81], MacLean, et al. [1992], or Thaler and Ziemba [1988] for further discussion.

To test for fair bets in the lottery, data on jackpot size, ticket sales, and game format was collected from 34 state and multi-state lotto games representing over 18,000 individual drawings. For each drawing, the w_i's and w_j can be calculated in straight forward manner based on the game matrix of the specific lotto, and dvr_t can be closely estimated using prevailing interest rates and the annuity length of the jackpot prize. The value of the lower-tier prizes is also available by examining the specific game rules, and the expected jackpot is widely advertised by lottery associations prior to each drawing.

A marginal tax rate of $\theta = 30$ percent was assumed for all drawings, although it must be noted that the nature of lottery players' behavior makes assigning a proper taxation rate difficult. First, marginal tax rates vary from state to state and some lottery winnings are exempt from state taxation. In addition, federal tax rates have varied (while generally declining) over the time period of the data. Next, most lottery players are unlikely to claim small prizes and losses on their tax returns, and lottery authorities are not required to report winnings of less than $600 to government tax agencies. Finally, winning a jackpot prize is very likely to cause a typical lottery player to move from a lower tax bracket to a higher one so that the marginal tax rate in equation (1) is not symmetric between winning and losing. There is no easy way around these difficulties, but the net effect of these influences makes it likely that a 30 percent tax rate overstates the effective tax rate for lower-tier prizes and for the tax deduction that players can take from the purchase of non-winning tickets, while perhaps slightly understating the tax rate when the jackpot prize is won. Obviously, the use of a different tax rate or a more complicated taxation structure would cause some of the drawings on the margin to change from positive to negative or vice versa, but the status of the vast majority of the drawings, both positive and negative, will be unchanged.

A true representation of the ex ante expected value of purchasing a lottery ticket requires that the player be able to make an accurate estimation of the number of other ticket buyers. In order to facilitate the examination of a large number of lotto games, this paper will instead examine the ex post expected return from the purchase of a lotto ticket based on actual ticket sales rather than buyer forecasted ticket sales. While it is certainly true that the ex post and ex ante ticket sales (and hence ex post and ex ante returns) may differ from one another if players inaccurately estimate ticket sales, previous research has found that players can quite closely estimate ticket sales and do not generally make systematic forecasting errors [Scott and Gulley, 1995; Matheson and Grote, 2003]. Given these results, it can be said that the ex ante and ex post estimates approximately match one another on any individual drawing and that on average over many drawings will exactly match. Indeed, some scholars would argue that equilibrium in the lotto market should be defined by the level of sales such that ex ante and ex post estimates exactly match one another. In any case, for simply ascertaining the relative frequency of fair bets in the lottery, the ex post method gives a good approximation with a significant reduction in computational difficulty.

TABLE 1
Expected Returns for Single Ticket purchase

Lottery	Dates of Data	Highest Observed Jackpot	Max. Expected Return (per $1.00 played)	# of Draws	# of Positive Draws	%Positive Draws
Multi-state "Powerball"	4/22/92 - 1/15/03	$315 million	$0.727	1,121	0	0%
Multi-state "Big Game"	9/06/96 - 5/04/99	$190 million	$0.776	215	0	0%
Tri-State "Megabucks"	3/12/97 - 5/29/99	$8.2 million	$0.719	214	0	0%
Tri-State "Win Cash"	9/12/97 - 5/28/99	$2.33 million	$0.973	179	0	0%
Tri-West "Lotto"	2/04/95 - 1/31/98	$1.63 million	$1.067	307	2	0.7%
Multi-state "Wild Card"	2/04/98 - 7/28/01	$2.06 million	$0.681	364	0	0%
Arizona "Lotto"	11/28/98 - 5/22/99	$10.1 million	$0.921	51	0	0%
California "Super Lotto"	10/18/86 - 1/19/02	$141 million	$0.753	1,544	0	0%
Colorado "Lotto"	9/14/90 - 7/28/01	$27 million	$0.977	1,150	0	0%
Connecticut "Lotto"	9/20/94 - 8/07/01	$26 million	$1.251	719	10	1.4%
Delaware "All Cash"	10/27/98 - 5/18/99	$1.13 million	$0.888	88	0	0%
Florida "Lotto"	5/07/88 - 7/28/01	$106.5 million	$0.945	783	0	0%
Georgia "Lotto"	8/31/96 - 8/04/01	$30.4 million	$1.027	258	1	0.4%
Illinois	4/14/99 - 8/01/01	$33 million	$1.253	241	6	2.5%
Indiana	9/03/94 - 8/01/01	$42 million	$1.292	542	9	1.7%
Kansas "Cash"	8/18/96 - 5/12/99	$2.00 million	$1.565	428	21	4.9%
Kentucky "Lotto"	3/01/95 - 7/28/01	$20 million	$1.444	670	29	4.3%
Louisiana	4/19/98 - 5/22/99	$2.05 million	$0.660	114	0	0%
Maryland	1/03/98 - 7/14/99	$18.5 million	$1.144	160	5	3.1%
Mass. "Megabucks"	11/05/97 - 8/11/01	$14.3 million	$1.340	394	21	5.3%
Mass. "Millions"	11/06/97 - 8/13/01	$30.6 million	$1.145	394	6	1.5%
Michigan "Lotto"	9/04/96 - 7/28/01	$40 million	$1.159	497	10	2.0%
Minnesota "Gopher 5"	5/24/91 - 7/24/01	$1.40 million	$0.918	1,062	0	0%
Missouri "Lotto"	1/03/96 - 6/30/01	$11.6 million	$1.546	459	22	4.8%
New Jersey	7/03/95 - 4/05/99	$35 million	$1.086	393	1	0.3%
New York	4/14/99 - 8/01/01	$45 million	$0.691	375	0	0%
Ohio "Super Lotto"	1/12/91 - 7/28/01	$54 million	$1.004	1,099	1	0.1%
Oregon "Lotto"	4/19/95 - 5/19/01	$18 million	$2.204	636	32	5.0%
Pennsylvania "Pick 6"	9/12/98 - 8/04/01	$73 million	$0.843	303	0	0%
South Dakota "Cash"	7/03/96 - 8/11/01	$0.34 million	$0.884	530	0	0%
Texas "Lotto"	11/14/92 - 1/15/03	$85 million	$0.969	1,061	0	0%
Virginia "Lotto"	1/27/90 - 5/05/99	$28 million	$1.168	929	6	0.7%
Washington	1/01/97 - 5/26/99	$24 million	$1.042	251	2	0.8%
Wisconsin	6/20/92 - 5/15/99	$16.5 million	$0.812	721	0	0%
Total				18,252	184	1.0%

Note: The "Highest Observed Jackpot" and "Max. Expected Return" may or may not correspond to the same drawing depending on factors such as the drawings' ticket sales, changes in prize structure, etc.

The results presented in Table 1 both confirm and counter the prevailing literature. Overall, it is shown that fair bets are indeed very rare occurrences with roughly 1 percent of drawings providing a player with a fair bet. This satisfies a common definition of rationality in gambling markets that games of chance should never present the player with a positive return. [e.g., Scott and Gulley, 1995]

On the other hand, the instances of fair bets may be significantly more common than previously believed. Half of the games studied showed at least one instance of a fair bet, and numerous games provided players with even odds on a relatively frequent basis. Several of the states exhibited even odds in 4 percent or more of the drawings.

It is particularly noteworthy that among the lotteries providing fair bets, several have maximum net expected payoffs well in excess of the price of the ticket with Indiana, Kansas, Kentucky and Missouri having a maximum expected gain of 40 percent or more and Oregon having a maximum expected return of over $2.20 on the purchase of a single one dollar ticket. These results conflict with widely held notions of efficiency in gambling markets in that persistent positive returns should induce a large increase in ticket purchases resulting in lower expected returns for each individual drawing and resulting in jackpots being more frequently won, eliminating long runs of positive drawings.

Another fact that can be observed in Table 1 is that lotteries with positive maximum expected payoffs tend to be in smaller states and particularly in states that offer both a state lotto game with lower average jackpots and a multi-state game with much larger jackpots. As hypothesized by Forrest, et al. [2002], players seem to react to big jackpots rather than big expected returns, again an apparent violation of rationality by ticket buyers.

This apparent violation of rationality is most easily explained by the concept of bounded rationality. For small gambles like a $1.00 lottery ticket, infrequent lotto players are likely to purchase tickets based upon limited information. In states with two games, media attention and in-store advertising is generally focused on the game offering the largest jackpot, and, therefore, it is much easier for a casual buyer to realize when this game's jackpot reaches an abnormally high level. The multi-state game thus attracts a large number of new buyers when the jackpot rises diminishing the expected value of the ticket. On the other hand, a relatively large increase in the jackpot of the state game may escape the attention of most players since even a record-high state game jackpot may still be smaller than a typical Powerball or Mega Millions prize. Therefore, players react to higher jackpots in the smaller state games with only slightly higher ticket purchases. Smaller games with potentially positive expected returns go largely unnoticed by many consumers in the shadow of nominally more impressive multi-state jackpots. States with independent games that do not participate in either of the big multi-state consortiums also experience few instances of positive expected returns. By similar reasoning, without another game to eclipse the advertised jackpot of the state game, ticket sales in these states are highly responsive to the jackpot, and bettors, therefore, "arbitrage away" any positive returns.

Another potential explanation for why jackpots rather than expected values seem to drive ticket sales in some games derives from the idea that a portion of the value of a lottery ticket is potentially non-monetary in nature, a hypothesis offered by many researchers to explain why otherwise risk averse consumers engage in gambling in the first place. A lotto drawing with a large jackpot has a high psychological value even if it has a low expected return so that the eye-popping jackpots offered by the largest lotto games attract large numbers of tickets buyers. This non-monetary value of large jackpots also explains why more bettors do not switch away from the multi-state game to the state game in the presence of positive expected returns in the smaller game. Dreaming about winning $100 million is more fun than dreaming about winning $5 million even if the smaller game offers a relatively better expected return.

Grote and Matheson [2006] go even further in exploring the notion of substitutability between lotto games. They find that in lottery associations offering both a state and a multi-state game, during periods of high jackpots (and therefore high expected returns) in the multi-state game, ticket sales increase for the state game as well. Rather than being substitutes, competing lotto games appear to be complements, further raising the question of efficiency and rationality in lotto markets.

Again, a psychological explanation is in order. The excitement generated by a large multi-state jackpot allows players to overcome any psychological or moral barriers they have to playing a game of chance. Once that barrier falls, the effective price of other gambling options such as the smaller lotto game also falls. In addition, since state and multi-state lottery tickets are generally sold in the same locations, a large increase in the number of multi-state ticket buyers increases the number of potential state lotto game buyers as well. These factors increase demand for the state lotto game and outweigh any substitution effects of regular state lotto game players temporarily switching away to the multi-state game.

EXPECTED PAYOFFS FROM THE "TRUMP TICKET"

It has been suggested that there may be conditions during which it may be profitable to corner a lottery game by purchasing every possible combination of numbers for a given drawing. Krautman and Ciecka [1993] and Matheson [2001] dub this strategy the "Trump Ticket." Estimating the expected payoffs requires some additional calculations. A good starting assumption is that other lottery players' decisions on whether to buy tickets remain constant regardless of whether another player buys the Trump Ticket.

Under this assumption, the purchase of a Trump Ticket does not affect the probability of any single ticket winning the jackpot nor does it change the expected number of winning tickets among the other buyers in the particular drawing. The purchase does, however, increase the size of the jackpot that the jackpot winner(s) receives. Since the purchase of the Trump Ticket necessitates a large purchase of tickets, if a specific portion of ticket sales is allocated to the jackpot prize pool, as in most games, the purchase of the Trump Ticket will cause a significant increase in the size of the jackpot. Mathematically, $AV_{jt}^{TT} = AV_{jt} + \tau\alpha_j dvr_t / w_j$ where AV_{jt}^{TT} is the advertised jackpot after the purchase of the Trump Ticket and α_j is the percentage of gross sales allocated to the jackpot pool. Since all number combinations are chosen under a Trump Ticket strategy, it is not necessary to assume that other players' number selections are uniformly distributed.

The issue of taxation again must be considered. As with the purchase of a single ticket, any winnings are fully taxable at the rate θ, but the Trump Ticket purchaser may deduct the cost of the tickets purchased to the extent of any winnings. If the purchaser's winnings exceed the cost of the Trump Ticket then the winnings less the cost of the Trump Ticket are taxable. If the purchaser's winnings are less than the cost of the Trump Ticket, then the full cost of the Trump Ticket is not deductible, but the purchaser will not have to pay taxes on any of the prizes, either.

TABLE 2
Expected returns for the Trump Ticket purchase

Lottery	Max. Expected Return: Single Ticket	Max. Expected Return: Trump Ticket	# of Draws	# of Positive Draws	%Positive Draws
Powerball	$0.727	$1.036	1,121	10	0.9%
Big Game	$0.776	$1.120	215	2	0.9%
Tri- Megabucks	$0.719	$1.114	214	3	1.4%
Tri- Win Cash	$0.973	$1.443	179	33	18.4%
Tri-West	$1.067	$1.590	307	67	21.8%
Wild Card	$0.681	$1.234	364	21	5.8%
Arizona	$0.921	$1.434	51	14	27.5%
California	$0.753	$1.109	1,544	7	0.5%
Colorado	$0.977	$1.397	1,150	91	7.9%
Connecticut	$1.251	$1.767	719	202	28.1%
Delaware	$0.888	$1.438	88	30	34.1%
Florida	$0.945	$1.321	783	23	2.9%
Georgia	$1.027	$1.368	258	28	10.9%
Illinois	$1.257	$1.745	241	43	17.8%
Indiana	$1.292	$1.812	542	86	15.9%
Kansas	$1.565	$2.055	428	93	21.7%
Kentucky	$1.444	$2.014	670	227	33.9%
Louisiana	$0.660	$0.982	114	0	0%
Maryland	$1.144	$1.545	160	45	28.1%
Mass Mega	$1.340	$1.764	394	87	22.1%
Mass Millions	$1.145	$1.630	394	145	36.8%
Michigan	$1.159	$1.488	497	60	12.1%
Minnesota	$0.918	$1.338	1,062	76	7.2%
Missouri	$1.546	$1.911	459	102	22.2%
New Jersey	$1.086	$1.531	393	27	6.9%
New York	$0.691	$1.043	375	3	0.8%
Ohio	$1.004	$1.281	1,099	48	4.4%
Oregon	$2.204	$2.498	636	96	15.1%
Pennsylvania	$0.853	$1.173	303	27	8.9%
South Dakota	$0.884	$1.330	530	34	6.4%
Texas	$0.969	$1.189	1,061	52	4.9%
Virginia	$1.168	$1.670	929	127	13.7%
Washington	$1.042	$1.305	251	19	7.6%
Wisconsin	$0.812	$1.360	721	84	11.7%
Total			18,252	2,012	11.0%

Table 2 shows the maximum expected return per dollar played for both a single ticket and a Trump Ticket purchase for every lotto game as well as the number of Trump Ticket drawings providing a fair bet. In comparing Tables 1 and 2, the first obvious conclusion is that Trump Ticket purchases are more often associated with positive expected returns than are single ticket purchases. As noted by Matheson [2001], under the assumption that other players' behavior is unchanged, the purchase of a Trump Ticket always has a higher expected return per dollar played than the purchase of a single ticket for two reasons. First, the purchase of the Trump Ticket increases the size of the jackpot without changing the expected number of other players matching the jackpot ticket leading to a higher expected payout from the grand prize. Second, because the purchase of the Trump ticket guarantees at least a share in the winning jackpot (as well as lower tier prizes), the purchaser of the Trump Ticket has a much higher chance of being able to deduct the price of the tickets from applicable taxes than the purchaser of a single ticket. Therefore, a significantly greater number of the lotteries studied provide opportunities for positive expected returns for the Trump Ticket purchaser than for the single ticket purchaser. With only one exception, each lottery examined shows at least one instance of the Trump Ticket providing greater than even odds.

The other startling aspect of Table 2 is simply the extraordinarily high number of times that the Trump Ticket presents a fair bet. Overall, 11 percent of the drawings examined provided an even odds bet for the purchase of the Trump Ticket with one-third of the games presenting a fair bet during at least 20 percent of draws. The size of the potential winnings is also surprising with many games offering an after-tax expected rate of return of over 50 percent at their highest point.

Of course, contrary to the primary assumption made in this analysis, other players' ticket buying behavior is, in fact, likely to be affected by the purchase of a Trump Ticket. The purchase of the Trump Ticket has the dual effect of simultaneously increasing the expected return to other players by increasing the size of the jackpot (since $AV_{jt}^{TT} = AV_{jt} + \tau\,\alpha_j\, dvr_t / w_j$) and reducing the expected return by increasing the number of expected winners sharing the jackpot by one. Equation (2) shows the pre-tax expected return from the jackpot portion of a lottery ticket when no Trump Ticket is purchased while equation (3) show the pre-tax expected return from the jackpot portion of a lottery ticket when a Trump Ticket is purchased.

$$
\begin{aligned}
(2)\qquad ER_t &= w_j\left(AV_{jt} / dvr_t\right)\sum_{x=0}^{B_t}(e^{-B_t w_j})(B_t w_j)^x / x!(1+x) \\
&= \left(AV_{jt} / dvr_t\right)(1-e^{-B_t w_j}) / B_t
\end{aligned}
$$

$$
(3)\qquad ER_t = w_j\left[\left(AV_{jt} / dvr_t\right) + \left(\tau\alpha_j / w_j\right)\right]\sum_{x=0}^{B_t}(e^{-B_t w_j})(B_t w_j)^x / x!(2+x)
$$

The expected return for other players from the purchase of a Trump Ticket rises as long as the jackpot rises faster than the number of expected winners. This occurs when the advertised jackpot is low or the number of other expected winners (besides the Trump Ticket) is high. Specifically, as the number of other ticket buyers approaches zero, the Trump Ticket increases the expected payout to other players if $AV_{jt} / dvr_t < \tau \alpha_j / w_j$ while as the number of other buyers becomes large, the Trump Ticket increases the expected payout to other players if $(AV_{jt} / dvr_t + \tau\alpha_j / w_j) / \alpha_j < B_t$.

Of course, a consortium will only find the purchase of the Trump Ticket profitable when a high jackpot is combined with a low number of other expected winners, so although the purchase of a Trump Ticket can either increase or decrease the expected return to other players, it is likely that anytime a Trump Ticket represents a profitable investment, the purchase of the Trump Ticket will result in a fall in the expected return to other players.

FIGURE 1
Breakeven points for Trump Ticket and Other Players

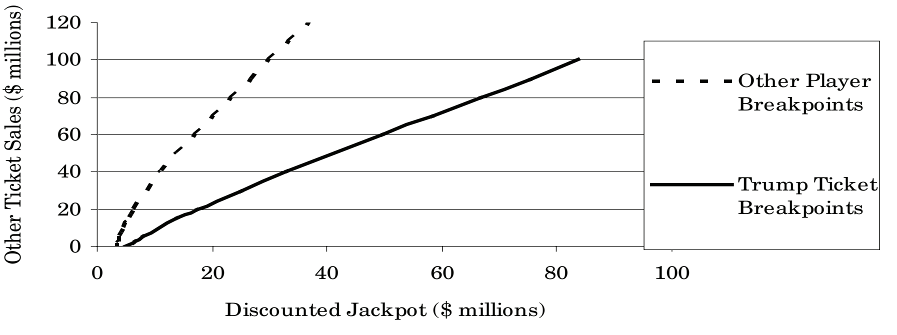

Figure 1 shows the combination of discounted jackpots and other ticket sales at which the net effect on the expected return to other players from the purchase of a single Trump Ticket is zero for a hypothetical lotto game with 10 million combinations, a 33.3 percent contribution to the jackpot fund, and a 16.7 percent contribution to lower-tier prizes. In addition, the combinations of discounted jackpots and other ticket sales representing the breakeven point for the Trump Ticket to be a fair bet is also shown. As hypothesized, for every level of ticket sales, the jackpot required to make the Trump Ticket a profitable gamble is well in excess of the jackpot level at which the purchase of the Trump Ticket begins to reduce the expected return for other players. For example, if there are 10 million other ticket buyers, the purchase of the Trump Ticket will result in a reduction in the expected return for these buyers as long as the drawing's discounted jackpot (before additions from the purchase of the Trump Ticket) is more than $4.64 million while a consortium will only find the purchase of the Trump Ticket profitable if the jackpot is greater than $10.88 million.

An examination of the observed levels of jackpots, ticket sales, and prize allocations in the 34 games examined in this study confirms this finding in all cases. Thus, in situations where a consortium would find the purchase of a Trump Ticket profitable, the expected return to the other players in the game would fall which should lead to a reduction in ticket purchases by these other players. Of course, if the number of other tickets purchased falls, since the jackpot is above the breakeven point for additional ticket sales to decrease expected return, the fall in the expected number of other winners is greater than the drop in the jackpot caused by these buyers exiting the market. Therefore, the expected return for the other players, including the purchaser of the Trump Ticket will rise. All in all, relaxing the assumption that ticket sales remain constant in the face of the purchase of a Trump Ticket would actually increase the number of observed draws in which the Trump Ticket provides a profitable gamble to the purchaser. In fact, the 11 percent figure quoted earlier as the proportion of drawings representing a fair bet to the purchaser of a Trump Ticket should be seen as a lower bound, and fair bets are likely to be even more common than this.

By similar reasoning, if a second Trump Ticket is sold during a particular drawing, given the assumption that the first Trump Ticket would only be purchased if it represented a fair bet, the expected value to other ticket buyers, including the purchaser of the other Trump Ticket buyer, would fall. Therefore, under a scenario where the purchase of a Trump Ticket is a simple transaction, buyers would attempt to make the purchase as soon as the gamble had a positive expected value, but such a purchase would have a negative expected value if a second buyer also appeared. One might well wonder how the process would unfold by which one but only one purchaser would step forward in the face of a fair bet.

The most obvious solution to this puzzle is to realize that even the purchase of a Trump Ticket, which guarantees a share of the jackpot, is still a risky investment. For example, consider the previously examined hypothetical lotto game with 10 million combinations, a 33.3 percent contribution to the jackpot fund, and a 16.7 percent contribution to lower-tier prizes. If there is a $12 million discounted jackpot (excluding the contributions from the Trump Ticket) and there are 10 million other ticket buyers (after accounting for changes in player behavior knowing that a Trump Ticket is being purchased), then the jackpot is $12 million plus one-third of $10 million or $15.33 million. The after-tax expected winnings from the purchase of a Trump Ticket is $10,586,635, a 5.9 percent return on a $10 million investment. However, the Trump Ticket results in a positive return only if no other winning tickets are sold in which case the player wins $15.33 million plus $1.67 million in lower tier prizes for a grand total of $17 million. If exactly one other winning ticket is sold, the purchaser of the Trump Ticket takes home only $9.33 million (half of $15.33 million plus the lower tier winnings), a 6.67 percent loss. With 10 million other ticket buyers, the purchaser of the Trump Ticket can count on being the sole winner only 36.8 percent of the time, so while the mean expected return is positive, the median return is negative.

With this understanding, as the jackpot of a lotto game grows, the expected value of the Trump Ticket rises. At some point above that at which the expected value turns positive, the least risk averse of the potential buyer consortiums will purchase the Trump Ticket. Once one purchaser steps forward, the fall in the expected value to other ticket buyers dissuades any further Trump Ticket purchases. Equilibrium is essentially a first-mover game, but the first-mover is defined as the least risk averse player.

FACILITATING THE PURCHASE OF THE TRUMP TICKET

If the purchase of a Trump Ticket consistently allows for supernormal rates of return, the real question is why one does not see attempts to corner lotto jackpots more often. The most likely reason is due to the transaction costs associated with the purchase of every number combination. The act of physically purchasing the 120 million combinations required for the Powerball Lottery, for example, is a daunting task. In fact, in February 1992, an Australian consortium attempted to corner a $25 million advertised jackpot in the Virginia Lotto. Despite a massive effort that included enlisting the aid of a major lottery ticket retailer, the consortium was only able to purchase 2.4 million of the 7,059,052 possible combinations before time ran out.

The obvious solution to this problem would be for lottery associations to allow for the direct purchase of a Trump Ticket by any buyer who can “write the check” for every combination. In theory, lottery associations could increase total ticket sales by allowing such a purchase. For example, the jackpot for the Mass Millions game reached levels that would have allowed for the profitable purchase of the Trump Ticket during two separate runs in 1998. The purchase of the Trump Ticket in both of these cases would have led to two one-time ticket sales of roughly $14 million each. Total ticket sales for the Mass Millions game were $64 million in 1998. Of course, the purchase of the Trump Ticket would eliminate the potential for high ticket sales in subsequent drawings since the jackpot would revert to its beginning value following the consortium winning the prize. We estimate that this factor would have reduced ticket sales by approximately $22 million. The result is a net increase in ticket sales by the Mass Millions game of roughly $6 million or 10 percent. Similarly, Oregon experienced four runs in 1998 where the jackpot exceeded the threshold for a profitable purchase of the Trump Ticket. The purchase of the Trump Ticket in each case would result in an additional $14 million in ticket sales countered by a corresponding drop of $6 million due to lower average jackpots. The $8 million in estimated gains would have represented a 25 percent increase in the actual ticket sales for Oregon.

In practice, no lottery association to our knowledge allows for such a direct purchase, and some take active steps to prevent cornering the jackpot. Many associations, for example, prohibit lotto terminal operators from purchasing tickets themselves. This would prevent a consortium from applying to operate a large number of lottery ticket machines and then using these machines to complete the purchase of a Trump Ticket themselves.

Lottery associations discourage the purchase of the Trump Ticket for several reasons. First, as noted previously, the purchase of the Trump Ticket would eliminate the potential for high ticket sales in subsequent drawings since the jackpot would revert to its beginning value following the consortium winning the prize. More importantly, however, the one-time ticket sale gains must be balanced against the possible loss of trust in the lottery by the public, who may feel that such a direct purchase is akin to “fixing” the lottery. In addition, if ticket sales are fueled by stories of the “regular guy” hitting it big, it is likely that stories of rich investment consortiums getting even richer through taking advantage of such a direct purchase may depress sales further. Still, a policy allowing a direct Trump Ticket purchase may be intriguing to lottery associations, which have generally experienced flat sales for many years.

CONCLUSIONS AND POLICY RECOMMENDATIONS

The results presented in this paper suggest that it is not only theoretically possible for lotteries to exhibit periods where the purchase of a single lottery ticket has a positive net present value, it is in fact a regular, though uncommon, occurrence for lotteries, especially in smaller states with more than one game, to exhibit this trait. Since the presence of a fair bet in the purchase of a single lottery ticket represents a violation of efficient markets, lottery associations where fair bets routinely occur should be able to increase ticket sales in the presence of these higher expected returns through public education and better advertising of high jackpots.

Gamblers should also take note that while the huge jackpots associated with the large multi-state games attract the most media attention and the frenzied buying, the best returns to players occur in the smaller games where relatively large jackpots do not spur "lotto fever" and the associated reduction in the expected values. It may be more exciting to dream about winning one of the huge "mega-jackpots" offered by the bigger lotteries than winning a "mere" couple of million, but the "smart" money is on taking a chance on the smaller jackpots.

In addition, it is extremely common that the purchase of the Trump Ticket, i.e. the purchase of all available combinations, would provide a fair bet to the buyer. Unfortunately, due to the difficulty of purchasing each individual number combination combined with the understandable reluctance of lottery associations to sell a Trump Ticket directly, these winning gambles will likely remain only hypothetical.

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