Is the Rent-Collector Worthy of His Full Hire?*

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The answer is No.

I. Statement of the Theorem

Allyn Young and Frank H. Knight have set straight the Marshall-Pigou fallacy that all increasing cost industries ought to be socially penalized: Young and Knight point out that the bidding up of rents to factors scarce to an industry are "transfer" costs to society; and, further, that such rents have to be charged if social efficiency is to be achieved. Otherwise land will get non-optimally utilized; fishing seas and roads may become overcrowded.

I take this all to be standard doctrine. Charging rents serves an efficiency purpose for a *laissez faire* or communistic society, even if we do not want any class called landlords to receive an income from rents.

The problem I pose here is this:

> Suppose that "landlords" are not made to give up any of their rents to laborers. Can it be true that the service these landlords create in improving the allocation of labor will be more than enough to pay their rent charges and still leave labor better off than before?

Were the answer to this Yes, labor would find it advantageous to vote unanimously for the institution of landlordism if the only alternative was that no rent would be charged by anyone. The present theorem states that Yes would be the wrong answer.

> Under the conditions postulated,[1] *the rent collected by landlords always represents more than the extra output society thereby achieves*; so in a certain sense, rent collection subject to no tax represents a subtraction (if not "exploitation") of labor.

I present this theorem not primarily for its interest as political economy. But it may have some implications for welfare analysis and it is a beautiful example where intermediate reasoning can establish what first appears to be a formidable problem viewed merely as cold mathematics.

* This paper was written in February 1962 but not published. In the meantime, Professor Martin Weitzman (then at Yale, now at M.I.T.) has independently originated a similar theorem and applied it brilliantly to the enclosure movement. Cf. Martin Weitzman and Jon S. Cohen, "A Marxian Model of Enclosures," July 31, 1972, unpublished. I owe thanks to the National Science Foundation for financial aid, and to Kate Crowley for editorial assistance.

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1 General diminishing returns and statical conditions—the latter because it is obvious that *laissez-faire* crowding of fishing waters could wipe out the last fish couple and *permanently* impoverish fishing labor.

II. A Test Example

Following in Robert L. Bishop's tradition at M.I.T. I set in January 1962 the following examination question for first-year graduate students.

> A village owns given amounts of grade A and grade B land. The average physical product schedules for corn of identical labor on the two types of land are given by
>
> $$APP_L^A = 8 - \frac{1}{2}L_A \text{ and } APP_L^B = 6 - \frac{1}{2}L_B.$$
>
> The total number of available identical labor is L = 6. The village uses the land as communal property dividing the lots to give each man an equal produce. Then at Ricardo's advice they charge rent (giving it back in an equal social bonus). Explain the reasons for this advice; derive the numerical value of the proposed rent; show in what way and by what amounts each villager will end up better off?

The good student, lending Ricardo Knight's insight, answers as follows. Prior to the collection of rents, labor overcrowded the good land, with equilibrium being at the point where average corn product was equalized on all lands used. (Five L_A and one L_B gave a wage rate of 5½ corn per worker and a total output for society of that much times the labor supply, or 33 corn in all.)

After the collecting of rent, the *marginal* rather than average labor productivities were equated: this relieved good land of the people who were there only because they could gain a slice of the high intra-marginal productivity such land would yield without them; the shift of labor to the poor land, where its marginal product was higher, does add to society's total output. (Arithmetically, with 4 and 2 for L_A and L_B, the wage is now at the equated marginal product level of 4 each per worker: the "residual rents" on the *A* and *B* land are 8 and 4 corn respectively. The total wage plus rent bill now adds up to 34, the increase over 33 being attributable to the efficiency aspects of rent collection on allocation. Note: though the total social pie has gone up, the amount that wage labor now ends up getting *as wages* has gone considerably *down*. This straight-line case therefore is one instance confirming my theorem that the total of rent always subtracts more from product than efficiency adds.)[2]

III. Literary Proof

We can begin with the case of but two plots of land, because that is simpler and because, fortunately, the case of any number of plots yields to essentially the same reasoning.

Before rent is charged, labor divides itself on the two plots so as to equalize their real wage rates at a common *average labor product* level. When the marginal products are unequal, it is evident we can add to total product by switching labor from the low to the high marginal-product areas. After land is appropriated by rent collectors, they will charge rent and ensure that the new configuration ends up with a common wage level at the now-equalized *marginal* products. Thus, in the new situation total output will have been increased by the new efficient allocation of labor. At the initial

[2] One student, regarding the initial situation as a problem of "fair division," supposed that initially each worker was allocated exactly one-sixth of A *and* one-sixth of B. Then if we can disregard waste-motion involved in working two plots, each will own his own small scale solve society's labor allocation problem. If he equalizes *marginal* productivities, technocratically or by charging *himself* rents, the optimum is achieved. This perhaps illustrates the efficiency merit of "private property" in the sense of providing *exclusive* use, without regard to rental pricing.

allocation of labor the respective *marginal labor products* will because of universal diminishing returns, each be less than their respective average products and hence less than the equalized wage level.

To compare the new and old wage rates, it is crucial to realize that the plot which has gained labor will, by the law of diminishing returns, end up with a lowered marginal product. But its initial marginal product was already shown to be less than the initial average-product wage rate. Hence, we have proved that the terminal marginal-product wage rate is definitely inferior to the initial average-product wage rate. This completes the proof that rent-collecting has definitely lowered pure wage income.[3]

The generalization to any number of land plots is fairly simple. First, we have the configuration yielded by the wage at equated average products; second, that yielded by the wage at equated marginal products. Except in the singular case so easily disposed of, there will be at least one plot of land that has had positive labor added to it. Hence, the proof from the two-plot case directly applies: On such a plot, the terminal marginal product wage is less than (or equal to) its initial marginal product, which is less than the initial average-product wage. *Q.E.D.*

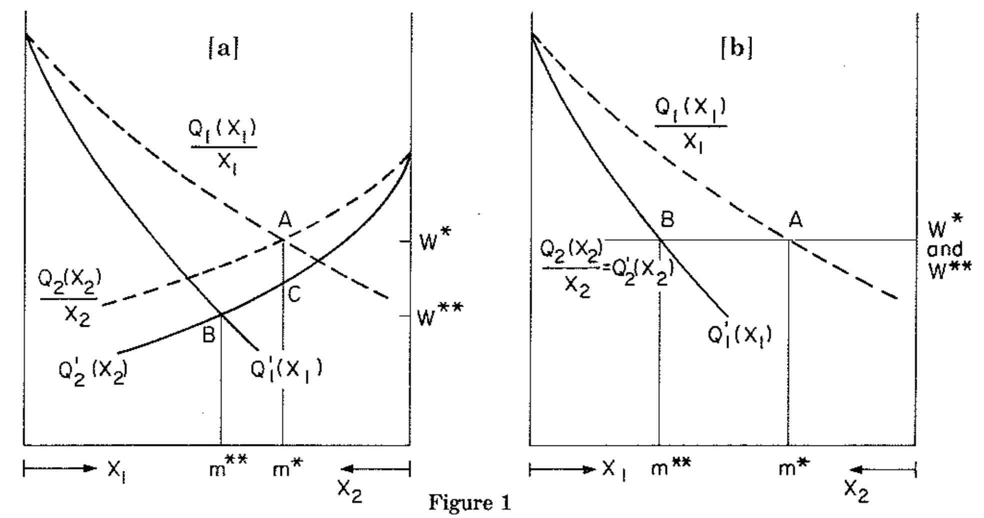

Figure 1

In Figure 1a, w°° at *A* is less than *B* which is less than *C*'s w°—as can be seen at a glance. [Figure 1b shows a borderline case in which the strong diminishing returns assumption has been relaxed on Land B, and where the rent triangle above B just matches the increment-of-total-social product triangle below BA.]

[3] The singular case, in which the equating of average products happens to coincide with the equating of marginal products, can be easily disposed of. In this case rent-collecting does not add to output at all; so, of course, collecting positive rents must lower wage incomes. (Alternative proof: Initial APP > Initial MPP = terminal MPP.) In the limiting case of everywhere-uniform-horizontal marginal productivity curves, there will of course be no rent under private property and no change in anything. That the ultimate loss in wages can be indefinitely large and the property return be grossly in excess of any function performed by the rent collector is shown by an example where the two plots have Cobb-Douglas functions with the same negligible exponent on labor, but with proportionality constants differing negligibly. In that case landlords get 99.9% of the product for achieving little or no improvement of the total product.

IV. Graphical Formulation and Proof

The good student presents the following graphical solution to the problem: First, he adds the average product curves horizontally; then he intersects the resulting curve with an inelastic labor supply schedule. The intersection gives him the common wage level and from the original individual curves he deduces the initial labor allocations and products. To get the terminal allocations, he adds the individual marginal product curves horizontally; again he finds the intersection of the vertical labor supply curve with this new aggregate curve, thereby getting the new common wage and so forth. (All this is rather like the Yntema-Robinson graphs for discriminating monopoly.)

For the two-good case, the Jevons-Wicksell diagram of Fig. 1 is even more convenient, and of course straight lines are not necessary.

V. Mathematical Formulation

To illustrate the power of economic intuition, one might present the following equivalent problem to an expert mathematician to see how rapidly he can solve it.

Theorem: We are given *n* non-negative strongly-concave monotone and smooth functions $Q_i(x)$ each with the property that $Q_i''(x) < 0$, $x > 0$. Then $w^{**} < w^{*}$, where

$$w^{*} = \frac{Q_1(x_1^{*})}{x_1^{*}} = \frac{Q_2(x_2^{*})}{x_2^{*}} = \ldots \frac{Q_n(x_n^{*})}{x_n^{*}} \quad , \qquad x_i > \quad ,$$

$$0 < x_1^{*} + \ldots + x_n^{*} = x = x_1^{**} + \ldots + x_n^{**} ,$$

and

$$w^{**} = Q_1'(x_1^{**}) = Q_2'(x_2^{**}) = \ldots = Q_n'(x_n^{**}) \quad , \quad x_i^{**} > 0.$$

The mathematical problem would look even more formidable if we replaced the above equalities by the following type of inequalities of modern programming type $w^{**} \geqslant Q_i'(x_i^{**})$, $x_i\{w^{**} - Q_i'(x_i^{**})\} = 0$, and so forth. Yet the theorem and proof would still be valid.

VI. Final Word

I draw no deeper welfare implications in this paper. Some may wish to note that here is one of the innumerable examples that can show the arbitrariness of those old new-welfare arguments which used to say: "If situation II could be better than situation III for everyone with proper compensating redistributions being made, then whether or not [!] such redistributions (or bribes) are made, society should overtly select II over I." Pareto-optimality is never enough.