

WHY DID MARSHALL TRANSPOSE THE AXES?

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Introduction

The teacher of elementary economics puts a schedule of prices and quantities demanded (or supplied) on the blackboard in which price is represented to be the independent variable and quantity the dependent one. He may follow this by writing the notation $Q = f(p)$. But then he draws a diagram in which quantity is put on the abscissa and price on the ordinate! Chances are that some student will challenge this, saying that the axes are wrong-way-round since mathematicians always put the independent variable on the abscissa. The lecturer might reply that, if the student will be patient, it will eventually be made plain that price and quantity are mutually dependent variables, just like the positions of three balls in a bowl, so it doesn't matter how we initially label the axes; or he might say that this will be fully explained in a more advanced class of microeconomics where the dynamics of market adjustment processes is examined; or he might simply say that it is just a matter of convention: mathematicians have their conventions and we economists have ours.

This graphic convention was established by Alfred Marshall, who used it in his lectures from which half the academic economists of England had learned their craft by the late 1800's (Foxwell, 1887, 92), and whose *Principles of Economics* taught even a larger number, and over a wider area, from its

publication in 1890 down to the second world war.¹

Prior to Marshall's *Principles*, Cartesian coordinate diagrams were not much used in economic analysis but such use as there was typically put price on the abscissa: Cournot (1838), Dupuit (1844), Lardner (1850), Jenkin (1870), Walras (1874-77).² F. Y. Edgeworth, not one of Marshall's students, in his Presidential Address to Section F of the British Association "On the Application of Mathematics to Political Economy" in 1889, presented a diagram of "Simple Exchange" in which the abscissa was price and the ordinate quantity, noting that "this is Cournot's construction. The converse construction in which the abscissa stands for quantity of commodity, the ordinate for price," Edgeworth added, "is employed by Mr. Wicksteed in his excellent *Alphabet of Economic*

¹The present writer was introduced to Marshall's *Principles* in his first course in economics, at Dalhousie University in 1941. This experience, while not common by then, was not unique: a student who took elementary economics at the University of Toronto in the same year recalls that Marshall's *Principles* was the text, though students having difficulty with it were advised to read F. W. Taussig's *Principles* instead.

²Walras consistently put quantity on the ordinate even when, in Lesson 8 of the *Elements*, he discussed the relationship between the quantity of a commodity and the utility derived from the consumption of it. In some diagrams the abscissa was labelled "rareté" which designated more or less what we now call marginal utility (see Walras, 1954, Jaffé's note [9], p. 506), with only a cryptic parenthetical remark calling the reader's attention to the fact that the ordinate is the independent variable in this case (p. 119).

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Science" (1925, II, 291).³ In 1894 Edgeworth accommodated to Marshall's (or Wicksteed's) practice, transposing the axes in his article on "Demand Curves" in *Palgrave's Dictionary of Political Economy* (Higgs, 1925, 542-544), and, by 1907, in reviewing a book by a French author, he felt it necessary to re-draw a diagram in the Marshallian form "so that the English reader may more readily apprehend the author's reasoning" (1925, III, 169).^{4,5}

Historians of economic thought agree that Marshall's *Principles* was responsible for the establishment of the current convention,⁶ but no cogent explanation has been advanced for Marshall's adoption of the practice. The object of this paper is to offer an explanation based upon an examination of Marshall's writings, paying particular attention to the purposes for which he used the theory of demand and supply.

In what follows I shall have to refer repeat-

³Wicksteed's *Alphabet* was published the preceding year, 1888. Edgeworth might also have mentioned Jevons, whose *Theory of Political Economy* (1871) contained numerous diagrams, all with quantity on the abscissa. Wicksteed and Jevons were both primarily interested in utility analysis, a point that is relevant, as we shall see, in understanding Marshall's diagrammatic practice.

⁴Edgeworth noted that the "Cournot-Dupuit-Colson system only seems preferable, so far as the amount demanded or supplied depends on the price," but acknowledged that Marshall's practice may have "a certain propriety" with respect to the long-run supply curve (1925, III, 169). It seems strange to me that Edgeworth, who discussed many technical matters with Marshall, apparently did not raise this point with him. At any rate, Edgeworth clearly did not understand Marshall's reasons for his choice of diagrammatic form.

⁵Schumpeter, in commencing his treatment of the Marshallian demand curve, states it verbally in terms of quantity being a function of price. He adds in a footnote: "Usually we put the independent variable, in this case the price, on the X-axis of a rectangular system of co-ordinates and the dependent variable, in this case the quantity, on the Y-axis. This is in fact usually done in the French literature. But Marshall chose the X-axis for quantity co-ordinate and the Y-axis for price co-ordinate, and this is usually done in the Anglo-American literature" (1954, 991). The customary practice of French economists has accommodated itself to the dominant Marshallian form only in recent years.

edly to the alternative ways of drawing demand and supply curves, expressing them algebraically, and discussing them verbally. To avoid tedium and achieve brevity, I shall refer to a diagram with price on the abscissa and quantity on the ordinate as being in the "quantity-function form," corresponding to the algebraic expression $Q = f(p)$. A diagram with quantity on the abscissa and price on the ordinate will be described as the "price-function form," corresponding to the expression $p = f(Q)$. The verbal counterpart of the quantity-function form describes a demand curve or function as depicting the maximum quantities consumers are willing to buy at various prices. For the supply curve or function, the quantity-function form depicts the maximum quantities (or the minimum quantities if the supply curve is negatively sloped) producers are willing to sell at various prices. The price-function form of a demand curve or function depicts the maximum prices consumers are willing to pay for various quantities, while the price-function form of a supply curve or function depicts the minimum prices producers are willing to accept for various quantities. It is my contention that one of the main reasons (perhaps the main reason) why Marshall adopted the price-function form was that he wished to interpret the price consumers are willing to pay as a measurement of utility and the price producers are willing to accept as a commensurable measurement of real (disutility) cost of production. In order to understand Marshall's diagrammatic practice we must pay attention to his normative welfare economics.

⁶The "convention" referred to applies, of course, only to diagrammatic practice. Algebraic notation usually represents quantity as a function of price, which has a long historical tradition. I note though that in Baumol and Goldfeld's anthology of precursors in mathematical economics (1968), Karl Schlesinger and Abraham Wald both expressed prices as functions of quantities and Pareto did the same in his rendition of Cournot's theory of monopoly. In Wald's celebrated 1936 paper his notation places price on the left of the equation sign.

Marshall's Practice

Due to Whitaker's excellent work on the Marshall papers we now have access to Marshall's unpublished writings on economics dating back almost⁷ to his earliest interest in the subject (*EEWAM*). The materials, which include such things as Marshall's annotations of Cournot's *Recherches* and a notebook of some early exercises in mathematical economics, enable one to study the origins and development of Marshall's economics much more fully than hitherto. These papers throw considerable light on the early development of Marshall's graphic method, his relation to predecessors, and the kinds of problems Marshall focused upon in using, and experimenting with, geometrical tools of analysis.

One of Marshall's first attempts to use graphic methods (if not the first) was a diagrammatic formulation of Ricardian rent theory which succeeded so well that Marshall noted that it "decided me to adopt curves as an engine" (*EEWAM*, 1, 40-41). In view of the similarity between the theory of rent and the theory of consumer and producer surpluses (which Marshall called "rents" in the early editions of the *Principles*) this graphic exercise has some relevance for the explanation I will offer of Marshall's decision to use the price-function form in graphing demand and supply curves.

So far as the materials in Whitaker's volumes enable one to determine, it appears that Marshall used the price-function form from the very beginning of his work on demand and supply. Especially notable are his notebook exercises on excise taxes and bridge tolls, written probably in the late 1860s or very early 70s, in which Marshall draws

demand-supply diagrams in the price-function form, employs the same form in the algebraic notation and, significantly, uses the analysis to measure the effects of taxes and tolls on consumer surplus (*Ibid.*, 2, 297-83).^{8,9}

At the time he made these notes, Marshall could not have been aware of Walras's work (1874-77) and their composition probably antedates Fleeming Jenkin's diagrams of supply and demand (1870). He could have known of Dupuit (1844) and Lardner (1850) at this point, but apparently did not.¹⁰ So the only graph-drawing predecessor known to Marshall, whose practice he had either to follow or reject at the time he settled on his own mode, was Cournot, (1838). Marshall studied Cournot's *Recherches* with great care, reading it for the first time around 1868 (*EEWAM*, 2, 240). The importance of Cournot to the development of his own economic ideas was clearly and generously acknowl-

⁸One might be tempted to argue that Marshall continued to use the price-function form because he happened to begin with it. But he was too good a mathematician to make such a decision without careful consideration and his development of the theory of consumer surplus in these notes indicates that his main reason for using the price-function form in the *Principles* was present in his earliest analytical work.

⁹Houghton (1958) objects to associating the idea of consumer surplus so strongly with Marshall, pointing out that Auspitz and Lieben published a well-worked out theory of it in 1889. But, by this date, Marshall's own analysis was twenty years old and he had lectured on it, and tutored students in it, during that period. Priority belongs, if to anyone other than Marshall, to Dupuit, from whom Auspitz and Lieben derived the foundations for their own analysis.

¹⁰Marshall's early analysis of bridge tolls (*EEWAM*, 2, 281-283) is so similar to Dupuit's (1844) that internal evidence alone would suggest that Marshall knew Dupuit's work at this time. All the other evidence, however, indicates that this is highly unlikely (*Ibid.*, 281 and n). In the first edition of the *Principles* Marshall included a footnote on authors who had used mathematical methods in economics; Jevons, Cournot, Gossen, Menger, Böhm-Bawerk, Walras, and Launhardt are mentioned, but not Dupuit or Lardner (*PE*, II, 247-248). Marshall's first references to Dupuit were made in the second edition of the *Principles* published in 1891.

⁷Marshall's earliest work in economics, undertaken shortly after his graduation from the Cambridge mathematics Tripos in 1865, consisted of reading Ricardo and Mill and translating their verbal theories and numerical illustrations into mathematics (*PE*, II, 6-8). Apparently no papers survive from this work. Whitaker's volumes begin with material from 1867.

edged in the preface to the first edition of the *Principles* (PE, I, ix-x).

In his algebra, Cournot expressed quantity as a function of price, except in his treatment of duopoly where he noted that "in this case it will be convenient to adopt the inverse notation $p = f(D)$ " (1960, 80).¹¹ In his diagrams, Cournot used the quantity-function form only.

Marshall's notes on Cournot (*EEWAM*, 2, 240-248) are revealing. In giving graphic renditions of Cournot's arguments Marshall uses both the quantity-function form (as Cournot had) and the price-function form, drawing three diagrams in the former mode and one in the latter. The main point of these notes is the geometric measurement and analysis of consumer and producer surplus, which Marshall carries out in *both* diagrammatic forms. These notes show then that the geometric analysis of consumer and producer surplus, and the use of it to assess welfare effects, was a main focal point of Marshall's analytic interest at least as early as his reading of Cournot; and that at the time he read, and made his notes on, Cournot's *Recherches*, Marshall worked comfortably in either function form, both algebraically and geometrically. Shortly thereafter apparently, he decided to use the price-function form exclusively in his own geometric work. Clearly, it would be confusing for an author's readers, if not for himself, if he drew supply-demand diagrams both ways, instead of choosing one mode and sticking with it. But the price-function form is the more useful of the two for some problems while the quantity-function form is more useful for others. Marshall decided, quite early apparently, to adopt the price-function form as his uniform geometrical and algebraic mode, but to vary the form

employed in verbal discussion as necessary for the purpose of the problem at hand.¹²

The adoption of such a practice requires that one inform the reader that a geometric demand or supply curve may be read both ways. So, for example, in the "primer book" that Marshall and his wife wrote for junior students one finds the initial statement of the "Law of Demand" worded as follows: "It is a matter of common experience that the larger the stock which sellers determine to sell, the lower will be the price at which it can be got rid of. *Vice versa*, the lower the price at which anything is offered for sale, the greater the amount of it which can be sold off" (EI, 69). Marshall undoubtedly learned from tutoring and examining students that this point required some emphasis. In the *Principles* there are many passages in which both function forms are stated alternately. In the initial discussions of demand, for example (PE, I, 96-100), he gives numerical schedules of prices and quantities, draws a demand curve, and in his verbal discussion shifts back and forth between the two modes several times in the space of four pages. In view of the care with which Marshall composed the *Principles* and the extensive revisions it was subjected to through eight editions, the only explanation which seems plausible is that Marshall regarded it necessary that the student of economics accustom himself to reading demand and supply curves both ways since they would have to be used in different forms for different purposes. There is one passage in the *Principles* in which he says that businessmen customarily think of supply in terms of the price at which a quantity can be sold (the

¹²In the Mathematical Appendix of the *Principles* Marshall consistently writes $y = f(x)$ as the demand equation, and $y = \phi(x)$ as the supply function, where x is quantity and y is price (PE, I, 838-858). He even uses this notation in Note III on elasticity where the verbal discussion is carried out in the quantity-function form (*Ibid.*, 839-840).

¹¹For a discussion of Cournot on this point, see Nichol (1934).

price-function form) and the demand in terms of sales which will be forthcoming at different prices (the quantity-function form) and notes that "economists commonly follow this practice" (PE, 457n), but obviously he was not recommending this in retaining the use of both forms of verbal analysis. On the contrary, he repeatedly emphasized the necessity of treating demand and supply in consistent and commensurable terms.

Why did Marshall transpose Cournot's axes? The answer, in my view, lies in Marshall's early, and continuing, interest in consumer and producer surplus,¹³ that is, in what we today call "normative welfare economics," as one of the main objects of economic analysis. There is no substantive difference, as such, in labelling coordinate axes one way or the other, but Marshall, in my opinion, put quantity on the abscissa and price on the ordinate because in doing the analysis of "maximum satisfaction" this would be *in accord with* the mathematical convention of using the abscissa for the independent, and the ordinate for the dependent, variables.

The "Doctrine of Maximum Satisfaction"

Whitaker's edition of Marshall's unpublished writings (*EEWAM*) shows that from the very beginning of his theoretical work on economics, Marshall interpreted the demand curve as showing the prices consumers are willing to pay, and the supply curve as showing the prices producers are willing to accept, for various quantities of a commodity. To map such curves in rectangular coordinates

¹³Note that I avoid the "apostrophe problem" by these expressions. Marshall always used an apostrophe with an s but he sometimes put the apostrophe before and sometimes after the s . It would be nice to record that his punctuation depended upon the aggregation implied in the analysis but, in fact, there was no consistency in his practice on this point. Samuelson is definitely incorrect in asserting that "with Marshall the apostrophe in consumers' surplus is always after the s " (1953, 206).

requires, if the geometric convention is obeyed, that quantity be measured, as the independent variable, on the abscissa, and price, the dependent variable, on the ordinate. Marshall's geometry, algebra, and verbal analysis were consistent and his diagrams were drawn in accordance with mathematical convention. The question, then, is not why Marshall labelled the axes as he did, but why he interpreted demand and supply as he did. He did so for two reasons, both of which are prominent in the *Principles* and are clearly present in his early analytical work of the late 1860's and early 70's.

One of these reasons has to do with Marshall's conception of how market adjustment processes work in the short and long-run. In the *Principles*, and his earlier work, Marshall pictures output as the equilibrating variable which brings about equality between the "demand price" and the "supply price." Cast in this form, the supply-demand diagram defines a stable equilibrium for a normal, negatively-sloped, demand curve, conjoined with a supply curve which is positively-sloped, horizontal, or negatively-sloped (as long as the latter's slope is not steeper than that of the demand curve). Marshall's market model may have developed from his early efforts to translate Ricardo and Mill into mathematics, the price-function form of the supply curve being a fairly obvious way of relaxing their assumption of constant cost. The classical practice of viewing price as the minimum compensation necessary to induce output, thus having been given a general formulation, it might have been natural for Marshall to define the demand curve in comparable price-function form and to interpret the market process in output adjustment terms.

Marshall's emphasis upon output adjustment in his diagrammatic treatment of demand and supply has been extensively discussed (see, e.g., Page, 1980), but it was

not the only reason why Marshall chose to use the price-function form and, in my opinion, not his *main* reason. This other reason has to do with the doctrine of "maximum satisfaction" which was the central pillar of the analytical edifice he commenced constructing in the late 1860s. The previous section emphasized that Marshall's early analytical efforts were directed at measuring consumer and producer surpluses, starting with a geometric representation of the Ricardian theory of rent. One of Marshall's notable innovations in economic theory was to use the concept of "surplus" in a very different way from his predecessors. The Physiocrats identified it ("*produit net*") with agriculture, and attributed it to the existence of free factors of production (sunlight, rain, etc.); the classical economists regarded surplus as the production above "subsistence," which was available for capital investment; Marx used the concept to define "exploitation" as labor's receiving less value than it produces. Marshall's surplus is a measure of the difference between the benefits accruing from productive effort and the costs incurred by it. Assuming a positive monotonic relationship between quantity of productive effort and quantity of output, costs and benefits can be construed as functions of output; so, in accordance with mathematical convention, output quantities should be represented on the abscissa and some index of benefits and costs on the ordinate. In Marshall's paradigmatic case of a boy picking blackberries (*PE*, I, 331, 844; *EEI*, 190) this is unproblematic since, for a single individual, benefits and costs can be regarded as measurable in commensurable terms—utilities and disutilities—and the boy's surplus is maximized when the marginal utility of blackberries (i.e., picking) equals the marginal disutility of picking (i.e., blackberries).

A great deal of Marshall's work was directed at extending the blackberries case to a world of division of labor and market exchange. Like Walras (1954, 256–257)

Marshall was totally dissatisfied with those who followed Bastiat in simply accepting intuitively "the famous doctrine that free competition tends to make the aggregate satisfaction a maximum" (*PE*, I, 470; II, 409).¹⁴ One of the main tasks of economic theory, in his view, was to formulate the theory of "maximum satisfaction" for an exchange economy in precise analytical terms so that its conditions could be clearly seen and its exceptions investigated. The lineal descendants of this line of Marshall's thought were the studies of exceptions by the next generation of Cambridge economists: Pigou on welfare economics, Robinson on imperfect competition, and Keynes on unemployment.^{15,16}

¹⁴This may have been the reason for Marshall's strange neglect of Henry Sidgwick's remarkable discussion of market failures (1883, Book III). He cannot have disagreed much with Sidgwick's propositions, but they were not erected upon solid analytical foundations. Marshall did not object to intuitive ethics, but science demanded the use of harder material. In his obituary of Sidgwick (1903), however, Marshall referred to this part of Sidgwick's *Principles* as "the best thing of its kind in any language" (*EEWAM*, I, 14n).

¹⁵In his comprehensive survey of utility theory from Adam Smith to E. E. Slutsky, George Stigler (1965) omits the application of utility theory to welfare economics on the ground that "most economists of the period used utility theory primarily to explain economic behavior (particularly demand behavior) and only secondarily (when at all) to amend or justify economic policy" (p. 67). He notes Marshall, Pigou, and some others, as exceptions (p. 97n).

¹⁶It is worth noting, without overarguing the significance of the point, that the Cambridge developments in imperfect competition theory and employment theory followed the Marshallian function form tradition in expressing value variables as functions of quantity variables. In Joan Robinson's theory of imperfect competition (1933) the crucial marginal revenue curve is derived directly from a demand curve in the price-function form and is itself formulated in analogous terms, with quantity as the independent variable. Keynes' formulation of the aggregate supply and aggregate demand functions in the *General Theory* are in the Marshallian form, expressing supply price and demand price as functions of the quantity of employment (1973, 25). The only diagram in the *General Theory*, however, maps the relationship of investment and the rate of interest in the quantity-function form. Keynes notes that this diagram was suggested by Harrod (1973, 180).

In examining this aspect of Marshall's work, it is important to recognize that he was not a Benthamite utilitarian, in either his ethics or his psychology. He did not believe that "pleasure" is the highest good, that people's wants should be regarded as data, that people are motivated only by self-interest, or that the efficiency of an economy can be determined by reference to the extent to which it satisfies currently perceived wants. On the contrary, the "Victorian moralizing" which sprinkles the pages of his analytical economics and punctuates his other writings, is distinctly non-utilitarian, contending that wants are products of cultural conditioning, that many of them are tawdry and ignoble, and that one of the main purposes of intellectual work (including theoretical economics) is to contribute to improvement in human character and the construction of a world suitable for, and inhabited by, better people.¹⁷ In this respect Marshall was more in agreement with the nineteenth-century romantics than the utilitarians and his frequent references to the "noble aspirations" of socialist thinkers reflected strong convictions which he held unchanged from early youth into old age.

How did such views harness with his scientific work and, specifically, with his doctrine of maximum satisfaction, which clearly seems to be based on a simple utilitarian calculus of benefits and costs? The answer is that, unlike the romantics and the socialists, Marshall believed that science, as hard as one could make it, is the essential foundation for social improvement. It was not the *end*, but it was the necessary *beginning*; the "mecca of economics," "economic biology," was a long way down a road which must be built patiently and carefully, one bit at a time. The

¹⁷See, for example, the appraisals of Marshall by Schumpeter (1941), Viner (1941), Shove (1942) and Levitt (1976). For a direct statement by Marshall on utilitarianism see *PE*, 17n. The leading economist of the post-Marshallian generation who most distinctively embraced a similar non-utilitarian conception of the good society was John Maurice Clark (1939, 1957).

first task was to construct a theory of surplus which would permit one to pass from the simple case of picking blackberries to the complex market economy where people are engaged in selling and buying instead of picking and eating. The analytical passage from the simple case to the complex one was made possible, in Marshall's view, by the role of money in an exchange economy, since the economist could employ money values, under certain conditions, as measurements of benefits and costs.

*Money Measurement*¹⁸

In 1896 Marshall addressed the first meeting of the newly-formed Cambridge Economic Club on the topic "The Old Generation of Economists and the New." One of his main themes was that the older generation had made great strides in *qualitative* analysis, but *quantitative* analysis, the *sine qua non* of applied science, had not yet made much headway.

Mere qualitative analysis . . . may show gain here and loss there; but it will not show whether the gain is sufficient to overbalance the loss; whether the gain should be pursued in spite of the loss. . . . It is useless to say that various gains and losses are incommensurable, and cannot be weighed against one another. For they must be, and in fact they are, weighed against one another before any deliberate decision is or can be reached on any issue (*MAM*, 301–302).

Fortunately, in Marshall's view, it was not only useless but unnecessary to regard benefits and costs as incommensurable. Twenty-years earlier he had noted that:

The pure science of Ethics halts for lack of a

¹⁸This section deals only with Marshall's use of money as a measuring unit for real benefits and costs, but it is also worth noting that Marshall's general monetary theory was based upon a demand curve for money as an asset (the cash-balance approach) which was formulated in the price-function form; i.e., the demand curve states the price (the amount of other things per unit) that people are willing to pay for various quantities of money. Marshall used this approach to monetary theory in his earliest work on the subject (see *EEWAM*, I, 164–177).

system of measurement of efforts, sacrifices, desires, etc., fit for her wide purposes. But the pure science of Political Economy has found a system that will subserve her narrower aims (*Ibid.*, 126).

That system is provided by money:

[T]he true philosophic *raison d'être* of [economic] theory is that it supplies a machinery to aid us in reasoning about those motives of human action which are measurable. In the world in which we live, money, as representing general purchasing power, is so much the best measure of motives that no other can compete with it (*Ibid.*, 158).¹⁹

In the opening chapters of the *Principles* Marshall discussed the characteristics of the modern economy and the springs of human action which lie behind market behavior, noting the play of religion, custom, collective sentiments and altruistic motives, but leading up through these considerations to a clear statement of the meaning of "Economic Generalization or Laws":

[A] law of social science, or a *Social Law*, is a statement of social tendencies; that is, a statement that a certain course of action may be expected under certain conditions from the members of a social group.

Economic Laws, or statements of economic tendencies, are those social laws which relate to branches of conduct in which the strength of the motives chiefly concerned can be measured by a money price (*PE*, I, 33).

Mary Marshall, recalling Alfred's early classes, noted that he regarded Bentham as having had "more influence on Economics than any other non-economist" due to his emphasis on measurement (*EEWAM*, 1, 11).²⁰ Concerning Adam Smith, Marshall

¹⁹In discussing this aspect of Marshall's thinking, Wesley C. Mitchell pointed out that Marshall regarded "motives" as conscious and rational matters, in contrast to the great amount of talk, by psychologists and others during his time, about instincts (1949, II, 68). Marshall's focus on rational action was also emphasized by Talcott Parsons (1949).

²⁰On Bentham's views on this point, see Goldworth (1979).

wrote:

His highest claim to have made an epoch in thought is that he was the first to make a careful and scientific inquiry into the manner in which value measures human motive, on the one side measuring the desire of purchasers to obtain wealth, and on the other the efforts and sacrifices (or "Real Cost of Production") undergone by its producers (*PE*, I, 758-759; see also *MAM*, 157).

Marshall's application of his views on money as a measure of motives is quite clear. In a demand-supply diagram, the ordinate is not a scalar of motives in themselves, or of (subjective) costs and benefits in themselves, but a *money measurement* of these. The justification for this construction is that if, in a free market, consumers are willing to pay as much as Y for a commodity, then they reveal by their actions that they value its benefits at equal to Y units of money; the demand curve is a money measurement of the marginal utility of the commodity at different rates of consumption. Similarly, for the supply curve, the ordinate is a money measurement of marginal costs at different rates of production. Benefits and costs now being measured in commensurable units, the difference between the two functions can be integrated: the area enclosed between the demand curve and the supply curve is a money measurement of the aggregate surplus of total benefits over total costs. In relation to Marshall's adoption of the price-function form, the significance of his view on the role of money measurement in economics is that price not only plays the part of a causal variable but serves as a common objective measuring unit of heterogeneous subjective entities. With respect to the latter role it belongs on the ordinate in a conventionally constructed diagram.

Any system of measurement must meet one fundamental requirement: the unit of reference must be invariant with respect to the quality being measured. A yardstick would not serve as a device for measuring (compara-

tive) lengths if it were inconstant in its own length. With respect to Marshall's unit of measure in his analysis of surplus, two problems are involved: the potential inconstancy of the marginal utility of money at different points on the supply or demand curves, and the potential differences in the marginal utility of money for different participants in market transactions, whether as consumers or producers. Marshall was acutely conscious of these two problems, and there are innumerable references in the *Principles*, and his other writings, to the necessity of taking them into account.²¹ In addition, if surplus is to be aggregated over commodities in order to make the "doctrine of maximum satisfaction" applicable to market processes in general, then the interdependence of commodities in consumer utility functions must be taken into account. Stigler notes that Marshall increasingly recognized the significance of commodity interdependence in successive editions of the *Principles* but that he only made "patchwork repairs" in his analysis and "retained to the last a theory constructed on the assumption of an additive utility function" (1965, 107).²² Economists have become equally dissatisfied with Marshall's handling of the constancy of the marginal utility of money.²³

Since the concern of this paper is to explain Marshall's diagrammatic practice, there is no need to evaluate his treatment of surplus or his use of money prices for measurement purposes. We should carry the study a step

²¹In general, Marshall treated the problem of holding the marginal utility of money constant along the demand and supply curves as a difficult, but not particularly important, *technical* problem, while he regarded the issue of differences in the marginal utility of money between persons as reflecting a very important social problem: the inequality of income distribution. Marshall had no doubt that the marginal utility of money is lower for the rich than for the poor, as a matter of *fact*, not merely presumption.

²²Marshall inserted a note into the third edition of the *Principles* saying that this problem can only be treated mathematically and added that such an attempted effort

further, however, and look more closely at Marshall's demand and supply curves in relation to the surplus analysis.

The Demand Curve and Consumer Surplus

The literature on this issue is so vast, and complex, that any brief comment is bound to be unsatisfactory. This section is restricted to providing some brief amplification of Marshall's concept of money measurement that is germane to an understanding of his use of the price-function form of the demand curve.

In Milton Friedman's celebrated paper on the Marshallian demand curve (1953), he argues that Marshall should be interpreted as drawing the demand curve on the assumption that real (not money) income is kept constant for all points on the curve. This means that as one moves from one point on the demand curve for a commodity X to another, the income effect of the price change is eliminated from the curve by assuming that some compensating change takes place which has the opposite effect on consumer real income. Two such compensating changes are considered by Friedman: a fall (rise) in money income as the price of X falls (rises); and a rise (fall) in other prices as the price of X falls (rises). Friedman interprets Marshall as adopting the latter compensating device, arguing that this harmonizes with one of Marshall's "basic organizing principles" in theory construction: the separation of the theory of relative prices from the theory of the

made "some years ago" by himself "convinced the present writer that even if the task be theoretically feasible, the result would be practically useless" (*PE*, I, 131n).

²³I disagree, however, with Corry's view that Marshall's welfare economics reflected his desire "to promote certain policy measures" (1968, 31), by which he is apparently referring to Marshall's support of income redistribution policies and his tax-bounty theorem on increasing and decreasing cost industries. Marshall's surplus analysis was derived too early in his thinking, and used too pervasively, to be attributed to such specific policy rationalizations.

general price level (1953, 65–68). But, elsewhere, Friedman acknowledges that Marshall seemed to be assuming compensating money income changes rather than compensating price changes (1953, 82, 92–94).

So far as Marshall's consumer surplus argument is concerned, the burden of evidence seems to be that Marshall assumed compensating money income changes in drawing up his demand curve. His favorite expository device was to picture a regime of perfect price discrimination, what Pigou later classified as discrimination of the "first degree" (1960, 279). In such a model, consumer real income is held constant in the demand curve for X by eliminating the income effect of any change in the price of X on intra-marginal units. Consumer surplus is then measurable by the excess of the integral of the prices that would be paid under price discrimination over the integral of uniform prices at the equilibrium point on a constant real income demand curve. Once again, we see Marshall's price-function form at work.

As Friedman notes (1953, 70), there is a problem here. Perfect price discrimination keeps real income constant, but it is not constant *at the level* defined by the market equilibrium; consumer surplus is consistently underestimated, due to holding real income constant at a level corresponding to a price of X so high that no X would be purchased. This is obviously unsatisfactory for goods of low income-elasticity, and especially so if one wishes to apply the consumer surplus analysis to a wide category of commodities such as "food."

Marshall seems to have been aware of this difficulty, but he did not consistently deal with it. He frequently referred to the consumer surplus analysis as being confined to individual commodities on which the expenditure is a small part of a consumer's budget (e.g., *PE*, I, 842). He noted, more generally, that "the ordinary demand and supply curves have no practical value except

in the immediate neighborhood of the point of equilibrium" (*Ibid.*, I, 384n; see also 131n, 133n), but he clearly did not stick to this in his applications of consumer surplus theory (see, e.g., the tax-bounty analysis discussed below) and, on one occasion at least, he *defined* consumer surplus as the loss that would be sustained if the commodity in question were not available at all (*Ibid.*, I, 830).

The Supply Curve and Producer Surplus

In the case of the boy picking blackberries, the supply curve can be interpreted as a mapping of disutilities incurred against quantities of blackberries produced. To translate this into a model of a market economy, the cost of production must be appropriately interpreted as a money measurement of disutility. This is the part of Marshall's economics in which he clung most tenaciously to his classical predecessors. He was reluctant to adopt the concept of opportunity cost (which was coming increasingly into vogue in the later nineteenth century) in part because it seemed to incorporate rent into cost of production (*PE*, I, 436–437n; II, 459–460), but more generally, because it failed to focus attention upon the disutility character of "real" costs.

In the primer book Marshall wrote with his wife and published eleven years before the *Principles*, the discussion of the "Law of Supply" is introduced in opportunity cost terms but then a formal definition is given and highlighted: "The Cost of Production of a thing . . . consists of the efforts and abstinences required for producing it" (*EI*, 73). Marshall retained this conception of cost throughout the remainder of his work, together with his early definition of "Expenses of Production" as the money measurement of these real costs in commensurable terms (*Ibid.*, 97). In the *Principles* he defined the supply curve in terms of the money compensation necessary to induce the quantities of labor and waiting required for different

quantities of production,²⁴ and repeatedly referred to the supply curve as a money measurement of such real costs (see, e.g., *PE*, I, 339, 352–353, 366). The only significant qualification Marshall made in this was that social real costs might well be higher than the "expenses of production" incurred by producers when young children are employed (*PE*, II, 412–413; *IT*, 183), though he also noted the more general point that the marginal utility of money might not be the same for all suppliers of factors (*EI*, 97).

On this basis Marshall was able to construct a theory of producer surplus that was homologous with his theory of consumer surplus.²⁵ The two surpluses taken together were a money measurement of the "satisfaction" which a well-functioning economy should maximize.²⁶

While this interpretation of Marshall's theory of producer surplus is amply supported by a comprehensive reading of his work, there are occasional remarks in his writings which seem to run counter to it. One remark which clearly seems to reject the treatment of the supply curve as measuring disutility cost is contained in a letter of 28 April 1892 to F. Y. Edgeworth:

I think Jevons did great harm by talking of supply-price as measuring disutility curve [sic]. In picking blackberries, the disutility curve of effort and the supply curve are practically the same thing and they are in *pari materia* with the demand curve or

the utility curve. But in the case of aneroid barometers, etc., the economic supply curve has but the slightest connection with the laws of disutility; for the greater part they are not in *pari materia* at all (*PE*, II, 811).

When interpreted in the light of Marshall's conception of the long-run supply curve, and his tax-bounty theorem, this will be seen to be perfectly consistent with the interpretation given above. Marshall used the aneroid barometers illustration in connection with production characterized by increasing returns due to economies of scale. In the same letter, Marshall added:

I have always held and taught in lectures year after year that Producer's rent cannot be represented in the supply curve except in curves in which you can ignore the economies of organization and production on a large scale (*Ibid.*, 812).

Marshall's treatment of economies of scale was cast entirely in the framework of his theory of long-run normal equilibrium so the explanation of his criticism of Jevons is to be found by noting the critical difference, in Marshall's view, between long-run and short-run supply curves. Before examining this directly, however, additional light may be supplied by noting Marshall's application of his surplus analysis to taxes and bounties on decreasing returns and increasing returns industries; that analysis is restricted to the long-run since, in Marshall's treatment, increasing returns is strictly a long-run phenomenon.

The tax-bounty theorem has frequently been cited as one of the most notable contributions of Marshall's *Principles* (see, e.g., Schumpeter, 1941, 245; Whitaker, in Marshall, *EEWAM*, 2, 343n). The essence of the theorem is that an excise tax, if levied on an increasing cost industry, will yield more revenue than the reduction in consumer surplus which results from its imposition, while an output subsidy, awarded to a decreasing cost industry, will cost less than the increase in

²⁴Curry, Murphy, and Schmitz (1971, 754) note that there is some confusion as to whether Marshall was referring to producing firms or original suppliers of production factors in his theory of producer surplus. They correctly interpret him as defining it in terms of factor suppliers but fail to recognize that Marshall's supply curve is a money measurement of real (disutility) cost.

²⁵Marshall divided producer surplus into a "worker's surplus" and a "saver's surplus" corresponding to the two types of real cost, making it plain that pure rent is a "surplus" of a quite different sort (*PE*, I, 830–832).

²⁶Stigler (1965, 83) points out that Marshall was clearly anticipated by H. H. Gossen in defining the maximum in such terms. Marshall referred to Gossen and Jevons in this connection starting with the third (1895) edition of the *Principles* (*PE*, II, 828).

consumer surplus which ensues. Thus, the intersection of free market demand and supply cannot be regarded as invariably defining optimum output since a judiciously designed mixture of taxes and bounties can alter outputs in such a way as to increase the aggregate consumer surplus.

Marshall's use of the consumer surplus device for tax analysis in ways foreshadowing the tax-bounty theorem of the *Principles* dates from his earliest work in economic theory in the late 1860's and early 1870's (see *EEWAM*, 2, 279–280; and 289–302). The theorem was pretty fully worked out in his manuscript on "The Pure Theory of Domestic Values" composed in the mid-1870's and privately printed by Sidgwick in 1879 (*Ibid.*, 223–236). The notes for Marshall's 1880 lectures at Bristol on "The Economic Influence of Government" made reference to similar propositions (*Ibid.*, 2, 384–385). In the *Principles* the tax-bounty theorem is contained in a chapter entitled "Theory of Changes in Normal Demand and Supply in Relation to the Doctrine of Maximum Satisfaction" where Marshall notes the case of decreasing cost industries as one of the two important qualifications to the proposition that the equilibrium of demand and supply defines the welfare optimum, the other being that the marginal utility of money is not the same to rich and poor consumers (*PE*, I, Bk. V, Ch. XIII).

The crucial point to note about Marshall's tax-bounty theorem is that the argument is carried out solely in terms of the effects on consumer surplus; producer surplus is not taken into account.²⁷ Why not? Producer surplus is included in Marshall's general discussion of the doctrine of maximum satisfaction; why is it absent in this case? The

²⁷Currie, Murphy, and Schmitz in their survey of the use of the surplus analysis in economic theory badly misinterpret Marshall on this point by confusing his concepts of pure rent and producer surplus (1971, 766–767, 773).

answer lies in the fact that the tax-bounty theorem was restricted to the circumstances where some industries operate under increasing cost conditions and others under decreasing cost conditions, but the latter can only be true of *long-run* cost. It is obvious that in the case of decreasing cost, if the cost curve is treated as mapping the first derivative of a total cost function against output, the integration of the function shows what, in Marshallian terms, would be a deficit rather than a surplus. But this was not Marshall's difficulty. In his view, the concept of producer surplus is inappropriate to long-run cost, whether increasing or decreasing.

Beginning with the second edition of the *Principles*, Marshall adopted the practice of referring to the short-run supply curve as a "particular expenses" curve. In his letter to Edgeworth quoted above, Marshall notes that he adopted the term "particular expenses" specifically to apply to supply curves from which an estimate of producer surplus may be derived. He never used the term in connection with the long-run supply curve. The main reason seems quite plain, though Marshall never stated it as explicitly as one would like: the short-run supply curve depicts the locus of *marginal* cost with some input factors held constant, while the long-run supply curve depicts the locus of *average* cost with all factors variable.²⁸

This simple point is not clearly expressed in the *Principles* because in Marshall's treat-

²⁸In the second edition of the *Principles*, in discussing the conditions of a stationary state, Marshall described the supply price as "the marginal price, the expectation of which just induced persons to enter the trade . . ." but deleted the word "marginal" from this passage in the fourth edition (*PE*, I, 367; II, 383). In another passage (*PE*, I, 373) dating from the second edition, (*PE*, II, 389), however, he refers to cost of production as "marginal" in both short and long periods as long as the aggregate volume of production (and hence the economies of scale) are held constant. Mishan (1968) has argued that Marshall's concept of producer surplus is inappropriate when applied to a long-run supply curve with all factors variable. He does not note that Marshall himself had similar reservations.

ment of long-run supply his main interest centered on external economies and the case of increasing return. In the clearest statement one can find on this he noted that any point on a long-run supply curve is drawn on the assumption that the representative firm has access to those internal and external economies it *would* enjoy if the aggregate production of the industry were *at the market equilibrium output*. The "particular expenses" at any point less than this would be higher than shown by the long-run supply curve (*PE*, I, 811–812). Where internal or external economies are such as to create increasing returns to scale, the long-run supply curve is especially inadequate as a depiction of marginal costs.

Marshall noted a further characteristic of the long-run supply curve which rendered it inappropriate for producer surplus analysis. His treatment of the long-run was cast in the framework of real time; hence, the supply function is not reversible. In moving to the left one does not lose all the economies of scale obtained at the higher output since some of these are due to knowledge, which is applicable to industrial organization for lower outputs. Thus, the locus of long-run costs traces out a lower curve for leftward movements (in real time) than rightward movements (*EEWAM*, 2, 202–204; 229; *PE*, 806n, 808n).

Marshall's analysis of the long-run and of increasing returns to scale was unsatisfactory. Marshall himself was not happy with it, recognizing it to involve, in an acute form, *time*: "the center of the chief difficulty of almost every economic problem" (*PE*, I, vii) and to require the extension of economic theory beyond statics to dynamics and, indeed, beyond dynamics to "biology" and the study of "economic evolution." In his discussion of long-run decreasing cost in Appendix H of the *Principles*, Marshall suggested the use of three dimensional geometric models with time as one of the variables, but he was,

in private, very critical of Cunyngame's efforts in this direction (*PE*, I, 809n; II, 808–810, 812). In a letter to Cunyngame on April 7, 1904, he said that "my case . . . of increasing returns never seemed to me of much practical use," noting also that the circumstances which make the long-run supply curve undefinable in real time also applied to the demand curve (*MAM*, 449).²⁹

Marshall's treatment of these matters was exceedingly unclear, but on the point at issue here he seems to be clear enough: the short-run supply curve can be treated as a static mapping of "particular expenses" against output, and therefore, it is a differential equation which can be integrated to show producer surplus. The long-period supply curve is not static, it does not unambiguously map "particular expenses," and its integral would be meaningless. These, it seems to me, were his grounds for rejecting Jevons' identification of the long-run supply curve with disutility cost.

Conclusion

A demand (supply) curve can be read two ways: as a statement of the quantities consumers (producers) will buy (sell) at different prices, or as a statement of the prices consumers (producers) are willing to pay (accept) for different quantities. According to the first reading, mathematical convention requires that quantity be treated as the dependent variable and price as the independent variable, and diagrammed accordingly,

²⁹In this letter to Cunyngame, Marshall was specifically referring to his analysis of tariffs and bounties in international trade. The point concerning the non-reversibility of the demand curve is equally damaging to his domestic tax-bounty theorem though Marshall did not give more than vague and passing acknowledgement of this (see *PE*, I, 807). Throughout his writings, Marshall displayed much greater doubt as to the wisdom of state intervention in international trade than in domestic markets. It is worth noting that Marshall's theory of economic development made heavy use of the concept of increasing returns in a domestic context (see, e.g., *IT*, 65, 106, 133; *MAM*, 266; also Stigler, 1941, 76).

but the second reading requires the opposite treatment. In this paper I have shown that Marshall interpreted demand (supply) functions in the second of these two ways from his earliest work in economic theory, and have argued that his treatment was, from the beginning and throughout his subsequent work, motivated by his desire to use price as a money measurement of benefits and costs in order to construct a theory of surplus serviceable for welfare economics. Thus, Marshall's diagrammatic practice was neither idiosyncratic nor mistaken, when viewed in terms of what he regarded to be the main purpose of economic theory.

References

- Baumol, William J. and Stephen M. Goldfeld, (eds.). *Precursors in Mathematical Economics: An Anthology*. London: London School of Economics and Political Science, 1968.
- Clark, John Maurice. *Economic Institutions and Human Welfare*. New York: Knopf, 1957.
- . *Social Control of Business*. New York: McGraw-Hill, 1939.
- Corry, Bernard. "Marshall, Alfred." David L. Sills (ed.), *International Encyclopedia of the Social Sciences*. Macmillan and Free Press, 1968, Vol. 10, pp. 25–33.
- Cournot, Augustin. *Researches into the Mathematical Principles of the Theory of Wealth* (1838). Translated by Nathaniel T. Bacon (1927). Reprinted New York: Kelley, 1960.
- Currie, John Martin, John A. Murphy, and Andrew Schmitz. "The Concept of Economic Surplus and Its Use in Economic Analysis." *Economic Journal*, 81, No. 324 (Dec. 1971): 741–799.
- Dupuit, Jules. "On the Measurement of the Utility of Public Works" (1844). Translated by R. H. Barback, *International Economic Papers*, No. 2 (1952): 83–110.
- Edgeworth, F. Y. *Papers Relating to Political Economy*. Three Volumes. London: Macmillan, 1925.
- Foxwell, H. "The Economic Movement in England." *Quarterly Journal of Economics*, II (Oct., 1887): 84–103.
- Friedman, Milton. "The Marshallian Demand Curve." *Journal of Political Economy*, LVII, No. 6 (December, 1949). Reprinted in *Essays in Positive Economics*. Chicago: University of Chicago Press, 1953, pp. 47–99.
- Goldworth, Amnon. "Jeremy Bentham: On the Measurement of Subjective States." *The Bentham Newsletter*, No. 2 (March 1979): 2–17.
- Higgs, Henry (ed.). *Palgrave's Dictionary of Political Economy*. London: Macmillan, 1925.
- Houghton, R. W. "A Note on the Early History of Consumer's Surplus." *Economica*, XXV, No. 97 (Feb., 1958): 49–57.
- Jenkin, Fleeming. "The Graphic Representation of the Laws of Supply and Demand and Their Application to Labour" (1870). Reprinted in *The Graphic Representation of the Laws of Supply and Demand, and Other Essays on Political Economy, 1886–1884*. London: London School of Economics and Political Science, 1931.
- Jevons, W. Stanley. *The Theory of Political Economy* (1871). Fifth edition edited by H. A. Jevons (1951), reprinted New York: Kelley and Millman, 1957.
- Keynes, John Maynard. *The General Theory of Employment Interest and Money* (1936). *The Collected Writings of John Maynard Keynes*, Vol. VII. London: Macmillan, 1973.
- Lardner, Dionysius. *Railway Economy* (1850). Reprinted New York: Kelley, 1968.
- Levitt, Theodore. "Alfred Marshall: Victorian Relevance for Modern Economics." *Quarterly Journal of Economics*, XC, No. 3 (August, 1976): 425–443.
- Marshall, Alfred. *The Early Economic Writings of Alfred Marshall, 1867–1890*. Edited by J. K. Whitaker. Two volumes. New York: Free Press, 1975. (Cited as *EEWAM*.)
- Marshall, Alfred. *Elements of Economics of Industry*. London: Macmillan, 1913 (First edition 1892). (Cited as *EEL*.)
- . *Principles of Economics*. Ninth (variorum) edition with Annotations by C. W. Guillebaud. Two volumes. London: Macmillan, 1961 (First edition 1890). (Cited as *PE*.)
- . *Industry and Trade*. New York: Macmillan, 1923. (Cited as *IT*.)
- Marshall, Alfred, and Mary Paley. *The Economics of Industry*. London: Macmillan, 1884 (First edition 1879). (Cited as *EI*.)
- Mishan, E. J. "What is Producer's Surplus?" *American Economic Review*, L, No. 4 (Dec., 1968): 1269–1282.
- Mitchell, Wesley C. *Lecture Notes on Types of Economic Theory*. Two volumes. New York: Kelley, 1949. (Mimeographed.)
- Nichol, A. J. "A Re-Appraisal of Cournot's Theory of Duopoly Price." *Journal of Political Economy*, 42, No. 1 (Feb. 1934): 80–105. Reprinted in Joseph J. Spengler and William R. Allen (eds.), *Essays in Economic Thought*. Chicago: Rand McNally, 1960, pp. 577–595.
- Page, Alfred N. "Marshall's Graphs and Walras' Equations: A Textbook Anomaly." *Economic Inquiry*, XVIII, No. 1 (Jan. 1980): 138–143.
- Parsons, Talcott. *The Structure of Social Action*. Glencoe, Ill.: Free Press, 1949.
- Pigou, A. C. *The Economics of Welfare*. Fourth edition (1932), reprinted with additions London: Macmillan, 1960.
- Robinson, Joan. *The Economics of Imperfect Competition* (1933). London: Macmillan, 1946.
- Samuelson, Paul Anthony. *Foundations of Economic Analysis*. Cambridge: Harvard University Press, 1953.
- Schumpeter, Joseph A. "Alfred Marshall's Principles: A Semi-Centennial Appraisal." *American Economic Review*, XXXI, No. 2 (June, 1941).
- . *History of Economic Analysis*. New York: Oxford University Press, 1954.
- Shove, G. F. "The Place of Marshall's Principles in the Development of Economic Theory." *Economic Journal*, LII, No. 208 (December, 1942): 294–329.
- Sidgwick, Henry. *The Principles of Political Economy*. (First edition London: Macmillan, 1883). Third edition (1901) reprinted New York: Kraus Reprint Co., 1969.
- Stigler, George. *Production and Distribution Theories: The Formative Period*. New York: Macmillan, 1941.
- . "The Development of Utility Theory." *Journal of Political Economy*, LVIII, Nos. 4 and 5 (August and October, 1950). Reprinted in *Essays in the History of Economics*, Chicago: University of Chicago Press, 1965, pp. 66–155.
- Viner, Jacob. "Marshall's Economics, In Relation to the Man and to His Times." *American Economic Review*, XXXI, No. 2 (June, 1941): 223–235.
- Wald, Abraham. "On Some Systems of Equations in Mathematical Economics" (1936). Translated by Otto Eckstein. *Econometrica* 19, No. 4 (Oct., 1951), 368–403.
- Walras, Léon. *Elements of Pure Economics, or The Theory of Social Wealth* (1874–77). Translated from edition of 1926 by William Jaffé. London: Allen and Unwin, 1954.
- Wicksteed, Philip H. *The Alphabet of Economic Science* (1888). Reprinted New York: Kelley and Millman, 1955.